IIT JAM Mathematical Statistics

Previous Year Questions by Syllabus Topic

JAM 2022–2026, 300 questions, mapped to the JAM 2027 syllabus

Coverage by syllabus topic

Counts use each problem's primary topic. Rows marked — have no PYQ in these papers.

Topic	2022	2023	2024	2025	2026	Total
1. Sequences and Series of Real Numbers						
1a Sequences: convergence, limits, Cauchy, monotone, limsup/liminf	2	1	1	2	2	8
1b Infinite series: tests for convergence, absolute/conditional convergence	1	2	2			5
1c Power series and radius of convergence			1	1	1	3
2. Differential and Integral Calculus						
2a One variable: limits, continuity, differentiability, Rolle/MVT, Taylor, L'Hospital	2	4	3	4	2	15
2b One variable: maxima, minima, points of inflection			1		1	2
2c Two variables: limits, continuity, partial and total derivatives	1	1	1		1	4
2d Two variables: maxima, minima, saddle points, Lagrange multipliers	2	2	1	1	1	7
2e Single integrals: FTC, Leibniz rule, improper integrals, Beta and Gamma	2	1	2	1		6
2f Double integrals, change of order, change of variables, areas/volumes/arc length	2	2	1	2	2	9
3. Matrices and Determinants						
3a Vector spaces: span, independence, basis, dimension, null space		1	1	1		3
3b Special matrices, determinants, trace, adjoint and inverse	3	1	2		2	8
3c Rank, nullity, row reduction, systems of linear equations	1	1	1	2	2	7
3d Eigenvalues, eigenvectors, Cayley–Hamilton	2	2	1	3		8
3e Quadratic forms and definiteness					1	1
4. Descriptive Statistics and Probability						
4a Descriptive statistics, correlation (simple, partial, multiple, rank)					3	3
4b Probability axioms, counting, inclusion–exclusion, geometric probability, Boole/Bonferroni	3	2	2	2	4	13
4c Conditional probability, total probability, Bayes, independence of events	2	2	3		2	9
5. Univariate Distributions						
5a Random variables, c.d.f./p.m.f./p.d.f., distribution of a function of a r.v.	2	1	3	1	3	10

Topic	2022	2023	2024	2025	2026	Total
5b Expectation, moments, quantiles, skewness/kurtosis, m.g.f.	3	3	5	4	2	17
5c Markov and Chebyshev inequalities	1		1		1	3
5d Standard discrete distributions	3	1	2	1		7
5e Standard continuous distributions		3			1	4
6. Multivariate Distributions						
6a Joint, marginal and conditional distributions; independence	1	3	1	3	1	9
6b Distribution of functions of random vectors (transformation, Jacobian)	1	1		2		4
6c Expectation of functions of random vectors, covariance, correlation, joint m.g.f.	2	1	3	2	1	9
6d Conditional expectation and conditional variance	1	2	1	1	2	7
6e Multinomial distribution	1	1				2
6f Bivariate normal distribution	2	1	1	3	1	8
7. Limit Theorems						
7a Modes of convergence and their inter-relations	1	1		1	1	4
7b WLLN, SLLN, Central Limit Theorem	1	3	4	2	2	12
8. Sampling Distributions						
8a Order statistics	3	1	3	2	2	11
8b Chi-square, t and F distributions; sampling distributions of statistics	3	3	3	4	2	15
9. Estimation						
9a Unbiasedness, consistency, relative efficiency		2	1	2		5
9b Sufficiency, factorization theorem, completeness				2	1	3
9c UMVUE, Rao–Blackwell, Lehmann–Scheffé, Cramér–Rao	1	2	1	2	1	7
9d Method of moments, maximum likelihood, invariance	3	2	2	3	3	13
9e Least squares and simple linear regression	1	1		1	1	4
9f Confidence intervals	1	1	1	1	1	5
10. Testing of Hypotheses						
10a Type I/II errors, size, power, p-value	5	3	2	1	3	14
10b Neyman–Pearson lemma, MP and UMP tests		1	2	3	1	7
10c Likelihood ratio tests	1	1	1		1	4
11. Nonparametric Methods						
11a Runs test, empirical d.f., Kolmogorov–Smirnov, sign tests, Mann–Whitney					2	2
12. Stochastic Processes						
12a Discrete-time Markov chains					2	2
12b Poisson process					1	1

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1 Sequences and Series of Real Numbers

1.1 1a. Sequences: convergence, limits, Cauchy, monotone, limsup/liminf

JAM 2022, Q.1 MCQ, 1 mark [1a]

Let $\{a_n\}_{n\geq 1}$ be a sequence of non-zero real numbers. Then which one of the following statements is true?

(A) If $\left\{\frac{a_{n+1}}{a_n}\right\}_{n\geq 1}$ is a convergent sequence, then $\{a_n\}_{n\geq 1}$ is also a convergent sequence

(B) If $\{a_n\}_{n\geq 1}$ is a bounded sequence, then $\{a_n\}_{n\geq 1}$ is a convergent sequence

(C) If $|a_{n+2} - a_{n+1}| \leq \frac{3}{4}|a_{n+1} - a_n|$ for all $n \geq 1$, then $\{a_n\}_{n\geq 1}$ is a Cauchy sequence

(D) If $\{|a_n|\}_{n\geq 1}$ is a Cauchy sequence, then $\{a_n\}_{n\geq 1}$ is also a Cauchy sequence

Answer: C

JAM 2022, Q.41 NAT, 1 mark [1a]

Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_{1+5m} = 2$, $a_{2+5m} = 3$, $a_{3+5m} = 4$, $a_{4+5m} = 5$, $a_{5+5m} = 6$, $m = 0, 1, 2, \ldots$. Then $\limsup_{n\to\infty} a_n + \liminf_{n\to\infty} a_n$ equals __________

Answer: 8

JAM 2023, Q.24 MCQ, 2 marks [1a]

Let $\{a_n\}_{n\geq 1}$ be a sequence such that $a_1 = 1$ and $4a_{n+1} = \sqrt{45 + 16a_n}$, $n = 1, 2, 3, \ldots$. Then, which one of the following statements is TRUE?

(A) $\{a_n\}_{n\geq 1}$ is monotonically increasing and converges to $\frac{17}{8}$

(B) $\{a_n\}_{n\geq 1}$ is monotonically increasing and converges to $\frac{9}{4}$

(C) $\{a_n\}_{n\geq 1}$ is bounded above by $\frac{17}{8}$

(D) $\sum_{n=1}^{\infty} a_n$ is convergent

Answer: B

JAM 2024, Q.1 MCQ, 1 mark [1a]

Let $a_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$ and $b_n = \frac{n^2}{2^n}$ for all $n \in \mathbb{N}$. Then

(A) $\{a_n\}$ is a Cauchy sequence but $\{b_n\}$ is NOT a Cauchy sequence

(B) $\{a_n\}$ is NOT a Cauchy sequence but $\{b_n\}$ is a Cauchy sequence

(C) both $\{a_n\}$ and $\{b_n\}$ are Cauchy sequences

(D) neither $\{a_n\}$ nor $\{b_n\}$ is a Cauchy sequence

Answer: B

JAM 2025, Q.1 MCQ, 1 mark [1a]

Let $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ be sequences given by

$$a_n = \left[\frac{n^2}{n+1}\right] \quad \text{and} \quad b_n = \frac{n^2}{n+1} - a_n.$$

Then

(A) $\{a_n\}_{n\geq 1}$ converges and $\{b_n\}_{n\geq 1}$ diverges

(B) $\{a_n\}_{n\geq 1}$ diverges and $\{b_n\}_{n\geq 1}$ converges

(C) Both $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ diverge

(D) Both $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ converge

Answer: B

JAM 2025, Q.31 MSQ (one or more correct), 2 marks [1a]

Let $\{x_n\}_{n\geq 1}$ be a sequence given by

$$x_n = \frac{2}{3}\left(x_{n-1} + \frac{2}{x_{n-1}}\right), \quad \text{for } n \geq 2,$$

with $x_1 = -10$. Then which of the following statement(s) is/are correct?

(A) $\{x_n\}_{n\geq 1}$ converges

(B) $\{x_n\}_{n\geq 1}$ diverges

(C) $x_{2025} - x_{2024}$ is positive

(D) $x_{2025} - x_{2024}$ is negative

Answer: A; C

JAM 2026, Q.11 MCQ, 2 marks [1a]

Let $f : \mathbb{N} \to \{1, 2, 3, \ldots, 100\}$ be an onto function. Consider the following statements.
$\boldsymbol{P}$: If $g(n) = \sup\{f(2n), f(2n+2), f(2n+4), \ldots\}$, $n \geq 1$, then

$$\lim_{n\to\infty} g(n) = \limsup_{n\to\infty} f(n).$$

$\boldsymbol{Q}$: If $h(n) = \inf\{f(n), f(n+1), f(n+2), \ldots\}$, $n \geq 1$, then

$$\lim_{n\to\infty} h(4n+1) = \liminf_{n\to\infty} f(n).$$

Then

(A) $\boldsymbol{P}$ is correct and $\boldsymbol{Q}$ is NOT correct

(B) $\boldsymbol{P}$ is NOT correct and $\boldsymbol{Q}$ is correct

(C) Both $\boldsymbol{P}$ and $\boldsymbol{Q}$ are correct

(D) Neither $\boldsymbol{P}$ nor $\boldsymbol{Q}$ is correct

Answer: B

JAM 2026, Q.31 MSQ (one or more correct), 2 marks [1a]

Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers given by

$$a_1 = 1, \quad a_{n+1} = \frac{a_n + 6}{a_n + 2}, \quad n \geq 1.$$

Which of the following statements is/are true?

(A) $a_{2024} - a_{2026} \geq 0$

(B) $\{a_n\}_{n\geq 9}$ is a monotone sequence

(C) $\{a_n\}_{n\geq 1}$ is a convergent sequence

(D) $a_n \leq 3$ for all n

Answer: A; C; D

1.2 1b. Infinite series: tests for convergence, absolute/conditional convergence

JAM 2022, Q.31 MSQ (one or more correct), 2 marks [1b]

Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_n = \frac{1}{3^n}$ for all $n \geq 1$. Then which of the following statements is/are true?

(A) $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ is a convergent series

(B) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}(a_1 + a_2 + \ldots + a_n)$ is a convergent series

(C) The radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$ is $\frac{1}{3}$

(D) $\sum_{n=1}^{\infty} a_n \sin\frac{1}{a_n}$ is a convergent series

Answer: A; B; D

JAM 2023, Q.25 MCQ, 2 marks [1b]

Let the series S and T be defined by

$$\sum_{n=0}^{\infty}\frac{2\cdot 5\cdot 8\cdots(3n+2)}{1\cdot 5\cdot 9\cdots(4n+1)} \quad\text{and}\quad \sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{-n^2},$$

respectively. Then, which one of the following statements is TRUE?

(A) S is convergent and T is divergent

(B) S is divergent and T is convergent

(C) Both S and T are convergent

(D) Both S and T are divergent

Answer: C

JAM 2023, Q.48 NAT, 1 mark [1b]

Let

$$S_n = \sum_{k=1}^{n}\frac{1+k\,2^k}{4^{k-1}}, \qquad n = 1, 2, \ldots.$$

Then, $\lim_{n\to\infty} S_n$ equals __________ (round off to two decimal places)

Answer: 9.32 to 9.34

JAM 2024, Q.11 MCQ, 2 marks [1b]

For $n \in \mathbb{N}$, let $a_n = \sqrt{n}\sin^2\left(\frac{1}{n}\right)\cos n$, and $b_n = \sqrt{n}\sin\left(\frac{1}{n^2}\right)\cos n$. Then

(A) the series $\sum_{n=1}^{\infty} a_n$ converges but the series $\sum_{n=1}^{\infty} b_n$ does NOT converge

(B) the series $\sum_{n=1}^{\infty} a_n$ does NOT converge but the series $\sum_{n=1}^{\infty} b_n$ converges

(C) both the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge

(D) neither the series $\sum_{n=1}^{\infty} a_n$ nor the series $\sum_{n=1}^{\infty} b_n$ converges

Answer: C

JAM 2024, Q.31 MSQ (one or more correct), 2 marks [1b]

Let $a_1 = 1$, $a_{n+1} = a_n\left(\frac{\sqrt{n}+\sin n}{n}\right)$ and $b_n = a_n^2$ for all $n \in \mathbb{N}$. Then which of the following statements is/are correct?

(A) the series $\sum_{n=1}^{\infty} a_n$ converges

(B) the series $\sum_{n=1}^{\infty} b_n$ converges

(C) the series $\sum_{n=1}^{\infty} a_n$ converges but the series $\sum_{n=1}^{\infty} b_n$ does NOT converge

(D) neither the series $\sum_{n=1}^{\infty} a_n$ nor the series $\sum_{n=1}^{\infty} b_n$ converges

Answer: A; B

1.3 1c. Power series and radius of convergence

JAM 2024, Q.41 NAT, 1 mark [1c]

The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{2n(x+3)^n}{5^n}$ is equal to __________ (answer in integer)

Answer: 5 to 5

JAM 2025, Q.42 NAT, 1 mark [1c]

The radius of convergence of the power series

$$\sum_{n\geq 1} \frac{n!}{n^n} x^n$$

is equal to ________
(round off to 2 decimal places)

Answer: 2.70 to 2.72

JAM 2026, Q.1 MCQ, 1 mark [1c]

If R denotes the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \left(\frac{2 \cdot 4 \cdot 6 \cdots 2n}{2 \cdot 5 \cdot 8 \cdots (3n-1)} \right)^2 x^n,$$

then which one of the following statements is true?

(A) $9R = 4$

(B) $4R = 9$

(C) $R = 1$

(D) $2R = 1$

Answer: B

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2 Differential and Integral Calculus

2.1 2a. One variable: limits, continuity, differentiability, Rolle/MVT, Taylor, L'Hospital

JAM 2022, Q.2 MCQ, 1 mark [2a]

Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} \lim\limits_{h\to 0} \dfrac{(x+h)\sin\left(\frac{1}{x}+h\right) - x\sin\frac{1}{x}}{h}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Then which one of the following statements is NOT true?

(A) $f\left(\frac{2}{\pi}\right) = 1$

(B) $f\left(\frac{1}{\pi}\right) = \frac{1}{\pi}$

(C) $f\left(-\frac{2}{\pi}\right) = -1$

(D) f is not continuous at $x = 0$

Answer: B

JAM 2022, Q.42 NAT, 1 mark [2a]

Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that

$$20(x-y) \leq f(x) - f(y) \leq 20(x-y) + 2(x-y)^2$$

for all $x, y \in \mathbb{R}$ and $f(0) = 2$. Then $f(101)$ equals __________

Answer: 2022

JAM 2023, Q.16 MCQ, 2 marks [2a]

For $x \in \mathbb{R}$, the curve $y = x^2$ intersects the curve $y = x\sin x + \cos x$ at exactly n points. Then, n equals

(A) 1

(B) 2

(C) 4

(D) 8

Answer: B

JAM 2023, Q.41 NAT, 1 mark [2a]

Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = x^2 - x$, $x \in \mathbb{R}$. Let $g : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that $g(x) = 0$ has exactly three distinct roots in the open interval $(0, 1)$. Let $h(x) = f(x)g(x)$, $x \in \mathbb{R}$, and h'' be the second order derivative of the function h. If n is the number of roots of $h''(x) = 0$ in $(0, 1)$, then the minimum possible value of n equals __________

Answer: 3 to 3

JAM 2023, Q.43 NAT, 1 mark [2a]

Let α and β be real constants such that

$$\lim_{x \to 0^+} \frac{\int_0^x \left(\frac{\alpha t^2}{1+t^4}\right) dt}{\beta x - \sin x} = 1.$$

Then, the value of $\alpha + \beta$ equals __________

Answer: 1.5 to 1.5

JAM 2023, Q.51 NAT, 2 marks [2a]

Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = x^2 \sin(x-1) + x\, e^{(x-1)}, \qquad x \in \mathbb{R}.$$

Then,

$$\lim_{n \to \infty} n \left(f\left(1 + \frac{1}{n}\right) + f\left(1 + \frac{2}{n}\right) + \cdots + f\left(1 + \frac{10}{n}\right) - 10 \right)$$

equals __________

Answer: 165 to 165

JAM 2024, Q.12 MCQ, 2 marks [2a]

Let $f_i\colon \mathbb{R}\to\mathbb{R}$, $i=1,2$, be defined by

$$f_1(x)=\begin{cases}\sin\left(\frac{1}{x}\right)+\cos\left(\frac{1}{x}\right), & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$$

and

$$f_2(x)=\begin{cases}x\left(\sin\left(\frac{1}{x}\right)+\cos\left(\frac{1}{x}\right)\right), & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}.$$

Then

(A) f_1 is continuous at 0 but f_2 is NOT continuous at 0

(B) f_1 is NOT continuous at 0 but f_2 is continuous at 0

(C) both f_1 and f_2 are continuous at 0

(D) neither f_1 nor f_2 is continuous at 0

Answer: B

JAM 2024, Q.32 MSQ (one or more correct), 2 marks [2a]

Let $f\colon\mathbb{R}\to\mathbb{R}$ be a twice differentiable function such that
$f(0)=0$, $f(2)=4$, $f(4)=4$ and $f(8)=12$.
Then which of the following statements is/are correct?

(A) $f'(x)\le 1$ for all $x\in[0,2]$

(B) $f'(x_1)>1$ for some $x_1\in[0,2]$

(C) $f'(x_2)>1$ for some $x_2\in[4,8]$

(D) $f''(x_3)=0$ for some $x_3\in[0,8]$

Answer: B; C; D

JAM 2024, Q.51 NAT, 2 marks [2a]

The value of $\lim_{n\to\infty} n\left(\sin\frac{1}{2n}-\frac{1}{2}e^{-\frac{1}{n}}+\frac{1}{2}\right)$ is equal to ___________ (answer in integer)

Answer: 1 to 1

JAM 2025, Q.11 MCQ, 2 marks [2a]

Let $f:\mathbb{R}\to\mathbb{R}$ be given by $f(x)=x+\pi\cos x$. Then the number of solutions of the equation $f(x)=0$ is

(A) 1

(B) 2

(C) 3

(D) 4

Answer: C

JAM 2025, Q.15 MCQ, 2 marks [2a]

A function $f:(0,1)\to\mathbb{R}$ is said to have property $\mathcal{I}$ if, for any $0<x_1<x_2<1$ and for any c between $f(x_1)$ and $f(x_2)$, there exists $y\in[x_1,x_2]$ such that $f(y)=c$. Consider the following statements:

(I) If $g:(0,1)\to\mathbb{R}$ satisfies property $\mathcal{I}$, then g is necessarily continuous.

(II) If $h:(0,1)\to\mathbb{R}$ is differentiable, then h' necessarily satisfies property $\mathcal{I}$.

Then

(A) (I) is correct and (II) is incorrect

(B) (I) is incorrect and (II) is correct

(C) Both (I) and (II) are correct

(D) Both (I) and (II) are incorrect

Answer: B

JAM 2025, Q.33 MSQ (one or more correct), 2 marks [2a]

Suppose $f : (0, \infty) \to (0, \infty)$ is continuously differentiable. Assume further that $\lim_{x\to\infty} f(x) = 0$. Which of the following statements is/are necessarily true?

(A) $\lim_{x\to\infty} f'(x)$ exists and is equal to 0

(B) $\limsup_{x\to\infty} f'(x) = 0$

(C) $\liminf_{x\to\infty} f'(x) = 0$

(D) $\liminf_{x\to\infty} |f'(x)| = 0$

Answer: D

JAM 2025, Q.43 NAT, 1 mark [2a]

Let

$$f(x) = x \sin\left(\frac{\pi}{2x}\right), \quad x > 0.$$

Then

$$\lim_{h\to 0} \frac{1}{\pi^2 h^2} [3f(1) - 2f(1+h) - f(1-2h)]$$

is equal to ________
(round off to 2 decimal places)

Answer: 0.75 to 0.75

JAM 2026, Q.12 MCQ, 2 marks [2a]

Let $h : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two bounded functions. Define the function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} xg(x) + x^2 h(x), & x > 0, \\ cx + dx^2, & x \leq 0, \end{cases}$$

where c, d are real constants. Consider the following statements.
$\boldsymbol{P}$: If $c = g(0) = h(0)$, then f is differentiable at $x = 0$.
$\boldsymbol{Q}$: If f, g, and h are thrice differentiable functions with $g(0) = 0$ and $g'(0) + h(0) > 0$, then f has a local minimum at $x = 0$.
Then

(A) $\boldsymbol{P}$ is correct and $\boldsymbol{Q}$ is NOT correct

(B) $\boldsymbol{P}$ is NOT correct and $\boldsymbol{Q}$ is correct

(C) Both $\boldsymbol{P}$ and $\boldsymbol{Q}$ are correct

(D) Neither $\boldsymbol{P}$ nor $\boldsymbol{Q}$ is correct

Answer: B

JAM 2026, Q.41 NAT, 1 mark [2a]

Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = 3e^x \cos 2x$. If $p(x) = ax^3 + bx^2 + cx + d$ is the third degree Taylor polynomial of f at $x = 0$, then $|a| + |b| + |c| + |d|$ equals __________ (in integer)

Answer: 16 to 16

2.2 2b. One variable: maxima, minima, points of inflection

JAM 2024, Q.14 MCQ, 2 marks [2b]

Let $f(x) = 4x^2 - \sin x + \cos 2x$ for all $x \in \mathbb{R}$. Then f has

(A) a point of local maximum

(B) no point of local minimum

(C) exactly one point of local minimum

(D) at least two points of local minima

Answer: C

JAM 2026, Q.2 MCQ, 1 mark [2b]

Let $g : \mathbb{R} \to \mathbb{R}$ be a function defined by $g(x) = \int_0^{x^3} (t^2 - 9t + 8)\, dt$. Which one of the following statements is NOT true?

(A) g has a local minimum at $x = 1$

(B) g has a point of inflection at $x = 0$

(C) $x = 0,\ x = 1,\ x = 2$ are critical points of g

(D) $g''(x) = 0$ has at least two distinct solutions, where g'' denotes the second order derivative of g

Answer: A

2.3 2c. Two variables: limits, continuity, partial and total derivatives

JAM 2022, Q.12 MCQ, 2 marks [2c]

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by

$$f(x,y) = \begin{cases} x^2 \sin \dfrac{1}{x} + y^2 \cos y, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Then which one of the following statements is NOT true?

(A) f is continuous at $(0,0)$

(B) The partial derivative of f with respect to x is not continuous at $(0,0)$

(C) The partial derivative of f with respect to y is continuous at $(0,0)$

(D) f is not differentiable at $(0,0)$

Answer: D

JAM 2023, Q.36 MSQ (one or more correct), 2 marks [2c]

Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x,y) = \begin{cases} \dfrac{xy(x+y)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Then, which of the following statements is/are TRUE?

(A) f is continuous on $\mathbb{R} \times \mathbb{R}$

(B) The partial derivative of f with respect to y exists at $(0,0)$, and is 0

(C) The partial derivative of f with respect to x is continuous on $\mathbb{R} \times \mathbb{R}$

(D) f is NOT differentiable at $(0,0)$

Answer: A; B; D

JAM 2024, Q.13 MCQ, 2 marks [2c]

Let $f(x,y) = |xy| + x$ for all $(x,y) \in \mathbb{R}^2$. Then the partial derivative of f with respect to x exists

(A) at $(0,0)$ but NOT at $(0,1)$

(B) at $(0,1)$ but NOT at $(0,0)$

(C) at $(0,0)$ and $(0,1)$, both

(D) neither at $(0,0)$ nor at $(0,1)$

Answer: A

JAM 2026, Q.14 MCQ, 2 marks [2c]

Let $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ and $g : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ be two bounded and differentiable functions. Define $F : \mathbb{R}^2 \to \mathbb{R}$ by

$$F(x,y) = \begin{cases} xf(y) + y^2 g(x), & xy \neq 0, \\ 0, & xy = 0. \end{cases}$$

Consider the following statements.
P: F is continuous at $(0,0)$.
Q: Both the partial derivatives $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$ exist at $(1,0)$.
Then

(A) ***P*** is correct and ***Q*** is NOT correct

(B) ***P*** is NOT correct and ***Q*** is correct

(C) Both ***P*** and ***Q*** are correct

(D) Neither ***P*** nor ***Q*** is correct

Answer: A

2.4 2d. Two variables: maxima, minima, saddle points, Lagrange multipliers

JAM 2022, Q.32 MSQ (one or more correct), 2 marks [2d]

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by

$$f(x,y) = 8(x^2 - y^2) - x^4 + y^4.$$

Then which of the following statements is/are true?

(A) f has 9 critical points

(B) f has a saddle point at $(2,2)$

(C) f has a local maximum at $(-2,0)$

(D) f has a local minimum at $(0,-2)$

Answer: A; B; C; D

JAM 2022, Q.51 NAT, 2 marks [2d]

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by $f(x,y) = x^2 - 12y$. If M and m be the maximum value and the minimum value, respectively, of the function f on the circle $x^2 + y^2 = 49$, then $|M| + |m|$ equals

Answer: 169

JAM 2023, Q.7 MCQ, 1 mark [2d]

For the function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by

$$f(x, y) = 2x^2 - xy - 3y^2 - 3x + 7y,$$

the point $(1, 1)$ is

(A) a point of local maximum

(B) a point of local minimum

(C) a saddle point

(D) NOT a critical point

Answer: C

JAM 2023, Q.30 MCQ, 2 marks [2d]

Among all the points on the sphere $x^2 + y^2 + z^2 = 24$, the point (α, β, γ) is closest to the point $(1, 2, -1)$. Then, the value of $\alpha + \beta + \gamma$ equals

(A) 4

(B) -4

(C) 2

(D) -2

Answer: A

JAM 2024, Q.2 MCQ, 1 mark [2d]

Let $f(x, y) = 2x^4 - 3y^2$ for all $(x, y) \in \mathbb{R}^2$. Then

(A) f has a point of local minimum

(B) f has a point of local maximum

(C) f has a saddle point

(D) f has no point of local minimum, no point of local maximum, and no saddle point

Answer: C

JAM 2025, Q.51 NAT, 2 marks [2d]

Let $C = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 12\}$ be a circle in the plane. Let (a, b) be the point on C which minimizes the distance to the point $(1, 2)$. Then $b - a$ is ________
(round off to 2 decimal places)

Answer: 1.54 to 1.56

JAM 2026, Q.13 MCQ, 2 marks [2d]

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = xy(5x + 3y - 6).$$

Which one of the following statements is NOT true?

(A) f has exactly 4 critical points

(B) f has more than one saddle point

(C) f has two local minima

(D) f has a critical point of the form (a, b) such that $a + b = \frac{16}{15}$

Answer: C

2.5 2e. Single integrals: FTC, Leibniz rule, improper integrals, Beta and Gamma

JAM 2022, Q.11 MCQ, 2 marks [2e]

$$\lim_{n\to\infty} \frac{6}{n+2} \left\{ \left(2+\frac{1}{n}\right)^2 + \left(2+\frac{2}{n}\right)^2 + \cdots + \left(2+\frac{n-1}{n}\right)^2 \right\}$$

equals

(A) 38

(B) 36

(C) 32

(D) 30

Answer: A

JAM 2022, Q.14 MCQ, 2 marks [2e]

Let $F : [0, 2] \to \mathbb{R}$ be the function defined by

$$F(x) = \int_{x^2}^{x+2} e^{x[t]}\, dt,$$

where $[t]$ denotes the greatest integer less than or equal to t. Then the value of the derivative of F at $x = 1$ equals

(A) $e^3 + 2e^2 - e$

(B) $e^3 - e^2 + 2e$

(C) $e^3 - 2e^2 + e$

(D) $e^3 + 2e^2 + e$

Answer: Marks to All

JAM 2023, Q.53 NAT, 2 marks [2e]

If, for some $\alpha \in (0, \infty)$,

$$\int_0^\infty 2^{-x^2}\, dx = \alpha\sqrt{\pi},$$

then α equals __________ (round off to two decimal places)

Answer: 0.59 to 0.61

JAM 2024, Q.15 MCQ, 2 marks [2e]

Consider the improper integrals

$$I_1 = \int_1^\infty \frac{t\sin t}{e^t}\, dt \quad \text{and} \quad I_2 = \int_1^\infty \frac{1}{\sqrt{t}} \ln\left(1+\frac{1}{t}\right) dt.$$

Then

(A) I_1 converges but I_2 does NOT converge

(B) I_1 does NOT converge but I_2 converges

(C) both I_1 and I_2 converge

(D) neither I_1 nor I_2 converges

Answer: C

JAM 2024, Q.42 NAT, 1 mark [2e]

Let $f(x) = \int_{-1}^{x^2-2x} e^{t^2-t}\,dt$ for all $x \in \mathbb{R}$. If f is decreasing on $(0, m)$ and increasing on (m, ∞), then the value of m is equal to __________ (answer in integer)

Answer: 1 to 1

JAM 2025, Q.3 MCQ, 1 mark [2e]

Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous odd function that is not identically zero. Further, suppose that f is a periodic function. Define

$$g(x) = \int_0^x f(t)\,dt.$$

Then

(A) g is odd and not periodic

(B) g is odd and periodic

(C) g is even and not periodic

(D) g is even and periodic

Answer: D

2.6 2f. Double integrals, change of order, change of variables, areas/volumes/arc length

JAM 2022, Q.13 MCQ, 2 marks [2f]

Let $f : [1, 2] \to \mathbb{R}$ be the function defined by

$$f(t) = \int_1^t \sqrt{x^2 e^{x^2} - 1}\,dx.$$

Then the arc length of the graph of f over the interval $[1, 2]$ equals

(A) $e^2 - \sqrt{e}$

(B) $e - \sqrt{e}$

(C) $e^2 - e$

(D) $e^2 - 1$

Answer: A

JAM 2022, Q.52 NAT, 2 marks [2f]

The value of

$$\int_0^2 \int_0^{2-x} (x+y)^2\, e^{\frac{2y}{x+y}}\,dy\,dx$$

equals __________ (round off to 2 decimal places)

Answer: 12.70 to 12.90

JAM 2023, Q.26 MCQ, 2 marks [2f]

The volume of the region

$$R = \{(x, y, z) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} : x^2 + y^2 \le 4,\ 0 \le z \le 4 - y\}$$

is

(A) $16\pi - 16$

(B) 16π

(C) 8π

(D) $16\pi + 4$

Answer: B

JAM 2023, Q.38 MSQ (one or more correct), 2 marks [2f]

Which of the following is/are TRUE?

(A) $\int_0^1 \int_0^1 e^{\max\{x^2,y^2\}}\,dx\,dy = e - 1$

(B) $\int_0^1 \int_0^1 e^{\min\{x^2,y^2\}}\,dx\,dy = \int_0^1 e^{t^2}\,dt - (e-1)$

(C) $\int_0^1 \int_0^1 e^{\max\{x^2,y^2\}}\,dx\,dy = 2\int_0^1 \int_y^1 e^{x^2}\,dx\,dy$

(D) $\int_0^1 \int_0^1 e^{\min\{x^2,y^2\}}\,dx\,dy = 2\int_0^1 \int_1^y e^{y^2}\,dx\,dy$

Answer: A; C

JAM 2024, Q.52 NAT, 2 marks [2f]

The value of the integral

$$\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{3\sqrt{x^2+y^2}}{\sqrt{8}\,\pi}\,dy\,dx$$

is equal to __________ (answer in integer)

Answer: 2 to 2

JAM 2025, Q.52 NAT, 2 marks [2f]

Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^3 + 2x^2 - 15x$ and $g(x) = x$ respectively. Let x_0 be the smallest strictly positive number such that $f(x_0) = 0$. Then the area of the region enclosed by the graphs of f and g between the lines $x = 0$ and $x = x_0$ is ________
(round off to 2 decimal places)

Answer: 33.70 to 33.80

JAM 2025, Q.53 NAT, 2 marks [2f]

Let V be the volume of the region

$$\left\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + \frac{z^2}{4} \leq 1 \text{ and } |z| \leq 1\right\}.$$

Then $\dfrac{V}{\pi}$ is equal to ________
(round off to 2 decimal places)

Answer: 1.82 to 1.84

JAM 2026, Q.15 MCQ, 2 marks [2f]

The bounded region enclosed by the curve $y = x^2 + 2$ and the line $y = 4x - 1$ is revolved about the x-axis to generate a solid. The volume of the generated solid equals

(A) $\frac{88}{5}\pi$

(B) $\frac{84}{5}\pi$

(C) $\frac{82}{5}\pi$

(D) $\frac{92}{5}\pi$

Answer: A

JAM 2026, Q.51 NAT, 2 marks [2f]

The area of the region in the first quadrant that is bounded by the parabola $y^2 = 36x$, the line $y = 3x - 9$ and x-axis equals __________ (in integer)

Answer: 54 to 54

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3 Matrices and Determinants

3.1 3a. Vector spaces: span, independence, basis, dimension, null space

JAM 2023, Q.2 MCQ, 1 mark [3a]

Let $M = M_1 M_2$, where M_1 and M_2 are two 3×3 distinct matrices. Consider the following two statements:
(I) The rows of M are linear combinations of rows of M_2.
(II) The columns of M are linear combinations of columns of M_1.
Then,

(A) only (I) is TRUE

(B) only (II) is TRUE

(C) both (I) and (II) are TRUE

(D) neither (I) nor (II) is TRUE

Answer: C

JAM 2024, Q.43 NAT, 1 mark [3a]

Let $V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 = x_2\}$. Consider V as a subspace of $\mathbb{R}^4$ over the real field. Then the dimension of V is equal to __________ (answer in integer)

Answer: 3 to 3

JAM 2025, Q.14 MCQ, 2 marks [3a]

Let V be a subspace of $\mathbb{R}^{10}$. Suppose A is a 10×10 matrix with real entries. Let $A^k(V) = \{A^k\mathbf{x} : \mathbf{x} \in V\}$ for $k \geq 1$ and $A(V) = A^1(V)$. Which one of the following statements is NOT true?

(A) If A is nonsingular, then $\dim(V) = \dim(A(V))$ necessarily holds

(B) It is possible that A is singular and $\dim(V) = \dim(A(V))$

(C) If $\operatorname{rank}(A) = 8$, then $\dim(A(V)) \geq \dim(V) - 2$ necessarily holds

(D) If $\dim(V) = \dim(A(V)) = \dim(A^2(V)) = \cdots = \dim(A^5(V))$, then $\dim(A^6(V)) = \dim(V)$ necessarily holds

Answer: D

3.2 3b. Special matrices, determinants, trace, adjoint and inverse

JAM 2022, Q.3 MCQ, 1 mark [3b]

Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \det \begin{pmatrix} 1+x & 9 & 9 \\ 9 & 1+x & 9 \\ 9 & 9 & 1+x \end{pmatrix}.$$

Then the maximum value of f on the interval $[9, 10]$ equals

(A) 118

(B) 112

(C) 114

(D) 116

Answer: D

JAM 2022, Q.33 MSQ (one or more correct), 2 marks [3b]

If $n \geq 2$, then which of the following statements is/are true?

(A) If A and B are $n \times n$ real orthogonal matrices such that $\det(A) + \det(B) = 0$, then $A + B$ is a singular matrix

(B) If A is an $n \times n$ real matrix such that $I_n + A$ is non-singular, then $I_n + (I_n + A)^{-1}(I_n - A)$ is a singular matrix

(C) If A is an $n \times n$ real skew-symmetric matrix, then $I_n - A^2$ is a non-singular matrix

(D) If A is an $n \times n$ real orthogonal matrix, then $\det(A - \lambda I_n) \neq 0$ for all $\lambda \in \{x \in \mathbb{R} : x \neq \pm 1\}$

Answer: A; C; D

JAM 2022, Q.43 NAT, 1 mark [3b]

Let A be a 3×3 real matrix such that $\det(A) = 6$ and

$$\operatorname{adj} A = \begin{pmatrix} 1 & -1 & 2 \\ 5 & 7 & 1 \\ -1 & 1 & 1 \end{pmatrix},$$

where $\operatorname{adj} A$ denotes the adjoint of A. Then the trace of A equals __________ (round off to 2 decimal places)

Answer: 3.45 to 3.55

JAM 2023, Q.31 MSQ (one or more correct), 2 marks [3b]

Let M be a 3×3 real matrix. If $P = M + M^T$ and $Q = M - M^T$, then which of the following statements is/are always TRUE?

(A) $\det(P^2 Q^3) = 0$

(B) $\operatorname{trace}(Q + Q^2) = 0$

(C) $X^T Q^2 X = 0$, for all $X \in \mathbb{R}^3$

(D) $X^T P X = 2X^T M X$, for all $X \in \mathbb{R}^3$

Answer: A; D

JAM 2024, Q.3 MCQ, 1 mark [3b]

Let $A = \begin{pmatrix} a & 0 \\ c & d \end{pmatrix}$ be a real matrix, where $ad = 1$ and $c \neq 0$. If

$$A^{-1} + (\operatorname{adj} A)^{-1} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix},$$

then $(\alpha, \beta, \gamma, \delta)$ is equal to

(A) $(a + d,\ 0,\ 0,\ a + d)$

(B) $(a + d,\ 0,\ c,\ a + d)$

(C) $(a,\ 0,\ 0,\ d)$

(D) $(a,\ 0,\ c,\ d)$

Answer: A

JAM 2024, Q.53 NAT, 2 marks [3b]

For some $a \leq 0$ and $b \in \mathbb{R}$, let

$$A = \begin{pmatrix} 0 & a & b \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}.$$

If A is an orthogonal matrix, then the value of $a\sqrt{6} + 4b\sqrt{3}$ is equal to __________ (answer in integer)

Answer: 2 to 2

JAM 2026, Q.16 MCQ, 2 marks [3b]

Let A be an $n \times n$ real matrix. Consider the following statements.
$\boldsymbol{P}$: If A is skew-symmetric, then $X^T AX = 0$, for every $n \times 1$ real matrix X.
$\boldsymbol{Q}$: If A is skew-symmetric and orthogonal, then n is even.
Then

(A) $\boldsymbol{P}$ is correct and $\boldsymbol{Q}$ is NOT correct

(B) $\boldsymbol{P}$ is NOT correct and $\boldsymbol{Q}$ is correct

(C) Both $\boldsymbol{P}$ and $\boldsymbol{Q}$ are correct

(D) Neither $\boldsymbol{P}$ nor $\boldsymbol{Q}$ is correct

Answer: C

JAM 2026, Q.52 NAT, 2 marks [3b]

Consider the real matrices

$$A = \begin{pmatrix} a & 3d & 3 \\ b & 3e & -2 \\ c & 3f & 1 \end{pmatrix}, \quad B = \begin{pmatrix} a & b & c \\ 1 & -3 & 5 \\ 2d & 2e & 2f \end{pmatrix}, \quad C = \begin{pmatrix} a & b & c \\ d & e & f \\ -1 & -4 & 9 \end{pmatrix}.$$

If the determinant of A is -48 and the determinant of B is 2, then the determinant of C equals __________ (in integer)

Answer: 14 to 14

3.3 3c. Rank, nullity, row reduction, systems of linear equations

JAM 2022, Q.15 MCQ, 2 marks [3c]

Let the system of equations

$$\begin{aligned} x + ay + z &= 1 \\ 2x + 4y + z &= -b \\ 3x + y + 2z &= b + 2 \end{aligned}$$

have infinitely many solutions, where a and b are real constants. Then the value of $2a + 8b$ equals

(A) -11

(B) -10

(C) -13

(D) -14

Answer: B

JAM 2023, Q.27 MCQ, 2 marks [3c]

For real constants α and β, suppose that the system of linear equations

$$x + 2y + 3z = 6; \qquad x + y + \alpha z = 3; \qquad 2y + z = \beta,$$

has infinitely many solutions. Then, the value of $4\alpha + 3\beta$ equals

(A) 18

(B) 23

(C) 28

(D) 32

Answer: C

JAM 2024, Q.16 MCQ, 2 marks [3c]

Let A be a 3×5 matrix defined by

$$A = \begin{pmatrix} 0 & 1 & 3 & 1 & 2 \\ 1 & 6 & 2 & 3 & 4 \\ 1 & 8 & 8 & 5 & 8 \end{pmatrix}.$$

Consider the system of linear equations given by

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 10 \end{pmatrix},$$

where x_1, x_2, x_3, x_4, x_5 are real variables. Then

(A) the rank of A is 2 and the given system has a solution

(B) the rank of A is 2 and the given system does NOT have a solution

(C) the rank of A is 3 and the given system has a solution

(D) the rank of A is 3 and the given system does NOT have a solution

Answer: B

JAM 2025, Q.13 MCQ, 2 marks [3c]

Let A be an $n \times n$ matrix. Which of the following statements is NOT necessarily true?

(A) If $\operatorname{rank}(A^5) = \operatorname{rank}(A^6)$, then $\operatorname{rank}(A^6) = \operatorname{rank}(A^7)$

(B) If $\operatorname{rank}(A) = n$, then it is possible to obtain a singular matrix by suitably changing a single entry of A

(C) If $\operatorname{rank}(A) = n$, then $\operatorname{rank}(A + A^T) \geq \dfrac{n}{2}$

(D) If $\operatorname{rank}(A) < n$, then it is possible to obtain a nonsingular matrix by suitably changing $n - \operatorname{rank}(A)$ entries of A

Answer: C

JAM 2025, Q.32 MSQ (one or more correct), 2 marks [3c]

Let $a, b \in \mathbb{R}$. Consider the system of linear equations

$$x + y + 3z = 5,$$
$$ax - y + 4z = 11,$$
$$2x + by + z = 3.$$

Then which of the following statements is/are correct?

(A) There are finitely many pairs (a, b) such that the system has a unique solution

(B) There are finitely many pairs (a, b) such that the system has no solution

(C) There are finitely many pairs (a, b) such that the system has infinitely many solutions

(D) If $a = b = 1$, the system has no solution

Answer: C

JAM 2026, Q.32 MSQ (one or more correct), 2 marks [3c]

Let A be an $m \times n$ real matrix with non-zero entries. Which of the following statements is/are true?

(A) If $m < n$, then there exists a non-zero $n \times 1$ real matrix X such that $AX = \mathbf{0}$, where $\mathbf{0}$ is the $m \times 1$ zero matrix

(B) If $m > n$, then $A^T A$ is a singular matrix

(C) If $n = 1$ and $m > 1$, then AA^T has nullity $m - 1$

(D) If there exists an $n \times m$ matrix B such that $AB = I_m$ and an $n \times m$ matrix C such that $CA = I_n$, then $n = m$

Answer: A; C; D

JAM 2026, Q.42 NAT, 1 mark [3c]

If the system of linear equations

$$x + y + z = 1$$
$$x + 2y - z = b$$
$$ax + 7y + z = b + 1,$$

where a and b are real constants, has infinitely many solutions, then $a + b$ equals __________ (in integer)

Answer: 3 to 3

3.4 3d. Eigenvalues, eigenvectors, Cayley–Hamilton

JAM 2022, Q.16 MCQ, 2 marks [3d]

Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$. Then the sum of all the elements of A^{100} equals

(A) 101

(B) 103

(C) 102

(D) 100

Answer: B

JAM 2022, Q.53 NAT, 2 marks [3d]

Let $A = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix}$ and let $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be an eigenvector corresponding to the smallest eigenvalue of A, satisfying $x_1^2 + x_2^2 + x_3^2 = 1$. Then the value of $|x_1| + |x_2| + |x_3|$ equals __________ (round off to 2 decimal places)

Answer: 1.70 to 1.76

JAM 2023, Q.1 MCQ, 1 mark [3d]

Let $M = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$. If a non-zero vector $X = (x, y, z)^T \in \mathbb{R}^3$ satisfies $M^6 X = X$, then a subspace of $\mathbb{R}^3$ that contains the vector X is

(A) $\{(x, y, z)^T \in \mathbb{R}^3 : x = 0,\ y + z = 0\}$

(B) $\{(x, y, z)^T \in \mathbb{R}^3 : y = 0,\ x + z = 0\}$

(C) $\{(x, y, z)^T \in \mathbb{R}^3 : z = 0,\ x + y = 0\}$

(D) $\{(x, y, z)^T \in \mathbb{R}^3 : x = 0,\ y - z = 0\}$

Answer: B

JAM 2023, Q.35 MSQ (one or more correct), 2 marks [3d]

Let P be a 3×3 matrix having the eigenvalues $1, 1$ and 2. Let $(1, -1, 2)^T$ be the only linearly independent eigenvector corresponding to the eigenvalue 1. If the adjoint of the matrix $2P$ is denoted by Q, then which of the following statements is/are TRUE?

(A) $\text{trace}(Q) = 20$

(B) $\det(Q) = 64$

(C) $(2, -2, 4)^T$ is an eigenvector of the matrix Q

(D) $Q^3 = 20Q^2 - 124Q + 256I_3$

Answer: A; C

JAM 2024, Q.33 MSQ (one or more correct), 2 marks [3d]

Let A be a 3×3 real matrix. Suppose that 1 and 2 are characteristic roots of A, and 12 is a characteristic root of $A + A^2$. Then which of the following statements is/are correct?

(A) $\det(A) \neq 0$

(B) $\det(A + A^2) \neq 0$

(C) $\det(A) = 0$

(D) trace of $(A + A^2)$ is 20

Answer: A; B; D

JAM 2025, Q.2 MCQ, 1 mark [3d]

Let a, b, c be real numbers with $b \neq c$. Define the matrix

$$M = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix}.$$

Then the number of characteristic roots of M that are real is

(A) 3

(B) 2

(C) 1

(D) 0

Answer: C

JAM 2025, Q.16 MCQ, 2 marks [3d]

Suppose f is a polynomial of degree n with real coefficients, and A is an $n \times n$ matrix with real entries satisfying $f(A) = 0$. Consider the following statements:

(I) If $f(0) \neq 0$, then A is necessarily nonsingular.

(II) If $f(0) = 0$, then A is necessarily singular.

Then

(A) (I) is correct and (II) is incorrect

(B) (I) is incorrect and (II) is correct

(C) Both (I) and (II) are correct

(D) Both (I) and (II) are incorrect

Answer: A

JAM 2025, Q.41 NAT, 1 mark [3d]

Let A be a 7×7 real matrix with $\mathrm{rank}(A) = 1$. Suppose the trace of A^2 is 2025. Let the characteristic polynomial of A be written as

$$\sum_{n=0}^{7} a_n x^n.$$

Then $\sum_{n=0}^{7} |a_n|$ is ________
(answer in integer)

Answer: 46 to 46

3.5 3e. Quadratic forms and definiteness

JAM 2026, Q.3 MCQ, 1 mark [3e]

Let A be a 2×2 real symmetric matrix such that the trace of A is 6 and the determinant of A is 5. Which one of the following statements is true?

(A) $A - I_2$ is positive definite

(B) $A - 3I_2$ is negative definite

(C) $A^2 + A - 3I_2$ is positive definite

(D) $A^2 - 6A$ is negative definite

Answer: D

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4 Descriptive Statistics and Probability

4.1 4a. Descriptive statistics, correlation (simple, partial, multiple, rank)

JAM 2026, Q.17 MCQ, 2 marks [4a]

Two different brands, namely Brand A and Brand B, of mobile batteries are tested and the following data on their lifetimes (in months) are obtained:

Brand A	27	28	30	32	33
Brand B	17	19	20	21	23

Let CV_A and CV_B denote the sample coefficients of variation for Brand A and Brand B, respectively. Let s_A and s_B denote the sample standard deviations for Brand A and Brand B, respectively. Which one of the following statements is true?

(A) $CV_A < CV_B$ and $s_A < s_B$

(B) $CV_A < CV_B$ and $s_A > s_B$

(C) $CV_A > CV_B$ and $s_A < s_B$

(D) $CV_A > CV_B$ and $s_A > s_B$

Answer: B

JAM 2026, Q.33 MSQ (one or more correct), 2 marks [4a]

The following data represent the lifetimes (in months) of a sample of eleven bulbs:

$$15,\ 10,\ 12,\ 10,\ 17,\ 12,\ 14,\ 20,\ 13,\ 22,\ 15$$

Which of the following statements is/are true?

(A) The sample mean is greater than the sample median

(B) The mean deviation about the point 15 equals 3

(C) The range of the data is twice the interquartile range of the data

(D) The sample standard deviation as well as the sample median are scaled by 2 if every data point is scaled by 2

Answer: A; B; D

JAM 2026, Q.43 NAT, 1 mark [4a]

In a singing competition, two judges award ranks for each of the 10 contestants. Based on the ranks awarded by the two judges, the Spearman's rank correlation coefficient ρ for the 10 contestants was computed to be 0.6. It was found later that the difference in ranks awarded by the two judges for one of the contestant was incorrectly taken as 2, instead of the correct value 5. If the correct difference is taken into account, then the corrected value of ρ equals __________ (rounded off to two decimal places)

Answer: 0.46 to 0.49

4.2 4b. Probability axioms, counting, inclusion–exclusion, geometric probability, Boole/Bonferroni

JAM 2022, Q.17 MCQ, 2 marks [4b]

Suppose that four persons enter a lift on the ground floor of a building. There are seven floors above the ground floor and each person independently chooses her exit floor as one of these seven floors. If each of them chooses the topmost floor with probability $\frac{1}{3}$ and each of the remaining floors with an equal probability, then the probability that no two of them exit at the same floor equals

(A) $\frac{200}{729}$

(B) $\frac{220}{729}$

(C) $\frac{240}{729}$

(D) $\frac{180}{729}$

Answer: A

JAM 2022, Q.34 MSQ (one or more correct), 2 marks [4b]

Let $\Omega = \{1, 2, 3, \ldots\}$ be the sample space of a random experiment and suppose that all subsets of Ω are events. Further, let P be a probability function such that $P(\{i\}) > 0$ for all $i \in \Omega$. Then which of the following statements is/are true?

(A) For every $\epsilon > 0$, there exists an event A such that $0 < P(A) < \epsilon$

(B) There exists a sequence of disjoint events $\{A_k\}_{k\geq 1}$ with $P(A_k) \geq 10^{-6}$ for all $k \geq 1$

(C) There exists $j \in \Omega$ such that $P(\{j\}) \geq P(\{i\})$ for all $i \in \Omega$

(D) Let $\{A_k\}_{k\geq 1}$ be a sequence of events such that $\sum_{k=1}^{\infty} P(A_k) < \infty$. Then for each $i \in \Omega$ there exists $N \geq 1$ (which may depend on i) such that $i \notin \bigcup_{k=N}^{\infty} A_k$

Answer: A; C; D

JAM 2022, Q.54 NAT, 2 marks [4b]

Five men go to a restaurant together and each of them orders a dish that is different from the dishes ordered by the other members of the group. However, the waiter serves the dishes randomly. Then the probability that exactly one of them gets the dish he ordered equals __________ (round off to 2 decimal places)

Answer: 0.35 to 0.40

JAM 2023, Q.8 MCQ, 1 mark [4b]

Let E_1, E_2 and E_3 be three events such that

$$P(E_1 \cap E_2) = \frac{1}{4}, \quad P(E_1 \cap E_3) = P(E_2 \cap E_3) = \frac{1}{5} \quad \text{and} \quad P(E_1 \cap E_2 \cap E_3) = \frac{1}{6}.$$

Then, among the events E_1, E_2 and E_3, the probability that at least two events occur, equals

(A) $\frac{17}{60}$

(B) $\frac{23}{60}$

(C) $\frac{19}{60}$

(D) $\frac{29}{60}$

Answer: C

JAM 2023, Q.13 MCQ, 2 marks [4b]

A point (a, b) is chosen at random from the rectangular region $[0, 2] \times [0, 4]$. Then, the probability that the area of the region

$$R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : bx + ay \leq ab,\ x, y \geq 0\}$$

will be less than 2, equals

(A) $\frac{1+\ln 2}{4}$

(B) $\frac{1+\ln 2}{2}$

(C) $\frac{2+\ln 2}{4}$

(D) $\frac{1+2\ln 2}{4}$

Answer: B

JAM 2024, Q.4 MCQ, 1 mark [4b]

A bag has 5 blue balls and 15 red balls. Three balls are drawn at random from the bag simultaneously. Then the probability that none of the chosen balls is blue equals

(A) $\frac{75}{152}$

(B) $\frac{91}{228}$

(C) $\frac{27}{64}$

(D) $\frac{273}{800}$

Answer: B

JAM 2024, Q.17 MCQ, 2 marks [4b]

Let $\Omega = \{1, 2, 3, 4, 5, 6\}$. Then which of the following classes of sets is an algebra?

(A) $\mathcal{F}_1 = \{\emptyset, \Omega, \{1, 2\}, \{3, 4\}, \{3, 6\}\}$

(B) $\mathcal{F}_2 = \{\emptyset, \Omega, \{1, 2, 3\}, \{4, 5, 6\}\}$

(C) $\mathcal{F}_3 = \{\emptyset, \Omega, \{1, 2\}, \{4, 5\}, \{1, 2, 4, 5\}, \{3, 4, 5, 6\}, \{1, 2, 3, 6\}\}$

(D) $\mathcal{F}_4 = \{\emptyset, \{4, 5\}, \{1, 2, 3, 6\}\}$

Answer: B

JAM 2025, Q.12 MCQ, 2 marks [4b]

A fair die is thrown three times independently. The probability that 4 is the maximum value that appears among these throws is equal to

(A) $\dfrac{8}{27}$

(B) $\dfrac{1}{216}$

(C) $\dfrac{37}{216}$

(D) $\dfrac{1}{2}$

Answer: C

JAM 2025, Q.50 NAT, 1 mark [4b]

A drawer contains 5 pairs of shoes of different sizes. Assume that all 10 shoes are distinguishable. A person selects 5 shoes from the drawer at random. Then the probability that there are exactly 2 complete pairs of shoes among these 5 shoes is equal to ________
(round off to 2 decimal places)

Answer: 0.23 to 0.25

JAM 2026, Q.18 MCQ, 2 marks [4b]

Consider the matrix

$$A = \begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix},$$

where X_{ij}, $1 \leq i, j \leq 3$, are independent and identically distributed random variables with the probability mass function

$$f(x) = \begin{cases} \frac{1}{3}, & x = -1, 0, 1, \\ 0, & \text{otherwise.} \end{cases}$$

The probability that A is orthogonal equals

(A) $\frac{2^4}{3^8}$

(B) $\frac{1}{3^5}$

(C) $\frac{2^5}{3^9}$

(D) $\frac{2^3}{3^7}$

Answer: A

JAM 2026, Q.34 MSQ (one or more correct), 2 marks [4b]

Let $S = \{1, 2, 3, \ldots\}$ and suppose that every subset of S is an event. Let $\mathcal{P}(S)$ denote the power set of S. Which of the following statements is/are true?

(A) There exists a probability function $P : \mathcal{P}(S) \to [0, \infty)$ such that $P(\{n\}) = \frac{1}{n+1}$, $n \geq 1$

(B) There exists a probability function $P : \mathcal{P}(S) \to [0, \infty)$ such that $P(A) = 0$ if A is a finite set and $P(A) = 1$ if A is an infinite set

(C) There exists a probability function $P : \mathcal{P}(S) \to [0, \infty)$ such that $P(\{1, 2, \ldots, n\}) = \int_1^n \frac{1}{x}\, dx$, $n \geq 1$

(D) There exists a probability function $P : \mathcal{P}(S) \to [0, \infty)$ such that $P(\{1, 2, \ldots, n\}) = \int_0^n e^{-x}\, dx$, $n \geq 1$

Answer: D

JAM 2026, Q.44 NAT, 1 mark [4b]

A point is chosen at random from a square region whose sides are of 2 metres. If the centre of the square is defined to be the point of intersection of its two diagonals, then the probability that the randomly chosen point is closer to the centre of the square than to any of its four corners equals __________ (rounded off to two decimal places)

Answer: 0.49 to 0.51

JAM 2026, Q.53 NAT, 2 marks [4b]

A fair six-sided die is rolled 4 times independently. If p is the probability that the sum of the outcomes is 14, then $10p$ equals __________ (rounded off to three decimal places)

Answer: 1.125 to 1.129

4.3 4c. Conditional probability, total probability, Bayes, independence of events

JAM 2022, Q.4 MCQ, 1 mark [4c]

Let A and B be two events such that $0 < P(A) < 1$ and $0 < P(B) < 1$. Then which one of the following statements is NOT true?

(A) If $P(A \mid B) > P(A)$, then $P(B \mid A) > P(B)$

(B) If $P(A \cup B) = 1$, then A and B cannot be independent

(C) If $P(A \mid B) > P(A)$, then $P(A^c \mid B) < P(A^c)$

(D) If $P(A \mid B) > P(A)$, then $P(A^c \mid B^c) < P(A^c)$

Answer: D

JAM 2022, Q.18 MCQ, 2 marks [4c]

A year is chosen at random from the set of years $\{2012, 2013, \ldots, 2021\}$. From the chosen year, a month is chosen at random and from the chosen month, a day is chosen at random. Given that the chosen day is the 29^{th} of a month, the conditional probability that the chosen month is February equals

(A) $\frac{279}{9965}$

(B) $\frac{289}{9965}$

(C) $\frac{269}{9965}$

(D) $\frac{259}{9965}$

Answer: A

JAM 2023, Q.47 NAT, 1 mark [4c]

Let the probability mass function of a random variable X be given by

$$P(X = n) = \frac{k}{(n-1)n}, \qquad n = 2, 3, \ldots,$$

where k is a positive constant. Then, $P(X \geq 17 \mid X \geq 5)$ equals __________

Answer: 0.25 to 0.25

JAM 2023, Q.49 NAT, 1 mark [4c]

A box contains a certain number of balls out of which 80% are white, 15% are blue and 5% are red. All the balls of the same color are indistinguishable. Among all the white balls, α% are marked defective, among all the blue balls, 6% are marked defective and among all the red balls, 9% are marked defective. A ball is chosen at random from the box. If the conditional probability that the chosen ball is white, given that it is defective, is 0.4, then α equals __________

Answer: 1.125 to 1.125

JAM 2024, Q.18 MCQ, 2 marks [4c]

Two fair coins S_1 and S_2 are tossed independently once. Let the events E, F and G be defined as follows:
E: Head appears on S_1
F: Head appears on S_2
G: The same outcome (head or tail) appears on both S_1 and S_2
Then which of the following statements is NOT correct?

(A) E and F are independent

(B) F and G are independent

(C) E and G^C are independent

(D) E, F, and G are mutually independent

Answer: D

JAM 2024, Q.34 MSQ (one or more correct), 2 marks [4c]

Consider four dice D_1, D_2, D_3, and D_4, each having six faces marked as follows:

Die	Marks on faces
D_1	$4, 4, 4, 4, 0, 0$
D_2	$3, 3, 3, 3, 3, 3$
D_3	$6, 6, 2, 2, 2, 2$
D_4	$5, 5, 5, 1, 1, 1$

In each roll of a die, each of its six faces is equally likely to occur. Suppose that each of these four dice is rolled once, and the marks on their upper faces are noted. Let the four rolls be independent. If X_i denotes the mark on the upper face of the die D_i, $i = 1, 2, 3, 4$, then which of the following statements is/are correct?

(A) $P(X_1 > X_2) = \frac{2}{3}$

(B) $P(X_3 > X_4) = \frac{2}{3}$

(C) $P(X_2 > X_3) = \frac{1}{3}$

(D) The events $\{X_1 > X_2\}$ and $\{X_2 > X_3\}$ are independent

Answer: A; B; D

JAM 2024, Q.54 NAT, 2 marks [4c]

Two factories F_1 and F_2 produce cricket bats that are labelled. Any randomly chosen bat produced by factory F_1 is defective with probability 0.5 and any randomly chosen bat produced by factory F_2 is defective with probability 0.1. One of the factories is chosen at random, and two bats are randomly purchased from the chosen factory. Let the labels on these purchased bats be B_1 and B_2. If B_1 is found to be defective, then the conditional probability that B_2 is also defective is equal to __________ (round off to 2 decimal places)

Answer: 0.42 to 0.44

JAM 2026, Q.5 MCQ, 1 mark [4c]

A retail shop sells three brands of tea, namely Brand A, Brand B, and Brand C, and each in two varieties, namely regular and premium. The proportion of customers buying the Brands A, B, and C are 40%, 35%, and 25%, respectively. Out of those customers who buy Brand A, 30% buy the premium variety, out of those who buy Brand B, 40% buy the premium variety, while out of those who buy Brand C, 60% buy the premium variety. Given that a randomly selected customer has bought the premium variety of tea, the probability that he/she has bought Brand B equals

(A) $\frac{14}{41}$

(B) $\frac{13}{31}$

(C) $\frac{15}{51}$

(D) $\frac{16}{61}$

Answer: A

JAM 2026, Q.19 MCQ, 2 marks [4c]

Let A, B, C be three events such that $P(C)P(C^c) > 0$. Consider the following statements.
$\boldsymbol{S1}$: $P(A|C) > P(B|C)$ and $P(A|C^c) > P(B|C^c)$ together imply that $P(A) > P(B)$.
$\boldsymbol{S2}$: $P(A|C) = P(A|C^c)$ implies that A and C are independent.
Then

(A) $\boldsymbol{S1}$ is correct and $\boldsymbol{S2}$ is NOT correct

(B) $\boldsymbol{S1}$ is NOT correct and $\boldsymbol{S2}$ is correct

(C) Both $\boldsymbol{S1}$ and $\boldsymbol{S2}$ are correct

(D) Neither $\boldsymbol{S1}$ nor $\boldsymbol{S2}$ is correct

Answer: C

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5 Univariate Distributions

5.1 5a. Random variables, c.d.f./p.m.f./p.d.f., distribution of a function of a r.v.

JAM 2022, Q.35 MSQ (one or more correct), 2 marks [5a]

A university bears the yearly medical expenses of each of its employees up to a maximum of Rs. 1000. If the yearly medical expenses of an employee exceed Rs. 1000, then the employee gets the excess amount from an insurance policy up to a maximum of Rs. 500. If the yearly medical expenses of a randomly selected employee has $U(250, 1750)$ distribution and Y denotes the amount the employee gets from the insurance policy, then which of the following statements is/are true?

(A) $E(Y) = \frac{500}{3}$

(B) $P(Y > 300) = \frac{3}{10}$

(C) The median of Y is zero

(D) The quantile of order 0.6 for Y equals 100

Answer: A; B; C

JAM 2022, Q.55 NAT, 2 marks [5a]

Let X be a random variable having the probability density function

$$f(x) = \begin{cases} ax^2 + b, & 0 \leq x \leq 3 \\ 0, & \text{otherwise,} \end{cases}$$

where a and b are real constants, and $P(X \geq 2) = \frac{2}{3}$. Then $E(X)$ equals __________ (round off to 2 decimal places)

Answer: 2.10 to 2.25

JAM 2023, Q.45 NAT, 1 mark [5a]

Let X be a random variable having the probability density function

$$f(x) = \begin{cases} \dfrac{x}{2}, & \text{if } 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Then, $\text{Var}\left(\ln\left(\frac{2}{X}\right)\right)$ equals __________

Answer: 0.25 to 0.25

JAM 2024, Q.6 MCQ, 1 mark [5a]

Let X be a continuous random variable having the $U(-2, 3)$ distribution. Then which of the following statements is correct?

(A) $2X + 5$ has the $U(1, 10)$ distribution

(B) $7 - 6X$ has the $U(-11, 19)$ distribution

(C) $3X^2 + 5$ has the $U(5, 32)$ distribution

(D) $|X|$ has the $U(0, 3)$ distribution

Answer: B

JAM 2024, Q.20 MCQ, 2 marks [5a]

Which of the following functions represents a cumulative distribution function?

(A) $F_1(x) = \begin{cases} 0, & \text{if } x < \frac{\pi}{4} \\ \sin x, & \text{if } \frac{\pi}{4} \leq x < \frac{3\pi}{4} \\ 1, & \text{if } x \geq \frac{3\pi}{4} \end{cases}$

(B) $F_2(x) = \begin{cases} 0, & \text{if } x < 0 \\ 2\sin x, & \text{if } 0 \leq x < \frac{\pi}{4} \\ 1, & \text{if } x \geq \frac{\pi}{4} \end{cases}$

(C) $F_3(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } 0 \leq x < \frac{1}{3} \\ x + \frac{1}{3}, & \text{if } \frac{1}{3} \leq x \leq \frac{1}{2} \\ 1, & \text{if } x > \frac{1}{2} \end{cases}$

(D) $F_4(x) = \begin{cases} 0, & \text{if } x < 0 \\ \sqrt{2}\sin x, & \text{if } 0 \leq x < \frac{\pi}{4} \\ 1, & \text{if } x \geq \frac{\pi}{4} \end{cases}$

Answer: D

JAM 2024, Q.55 NAT, 2 marks [5a]

Let X be a discrete random variable with $P(X \in \{-5, -3, 0, 3, 5\}) = 1$. Suppose that

$$P(X = -3) = P(X = -5),$$

$$P(X = 3) = P(X = 5) \quad \text{and}$$

$$P(X > 0) = P(X = 0) = P(X < 0).$$

Then the value of $12P(X = 3)$ is equal to __________ (answer in integer)

Answer: 2 to 2

JAM 2025, Q.20 MCQ, 2 marks [5a]

Let X be an $\text{Exp}(\lambda)$ random variable. Suppose $Y = \min\{X, 2\}$. Let F_X and F_Y denote the distribution functions of X and Y respectively. Then which of the following statements is true?

(A) F_Y is a continuous function

(B) $F_Y(y)$ is discontinuous at $y = 2$

(C) $F_Y(t) \leq F_X(t)$ for all $t \in \mathbb{R}$

(D) $\mathbb{E}(Y) > \mathbb{E}(X)$

Answer: B

JAM 2026, Q.4 MCQ, 1 mark [5a]

Let X be a positive continuous random variable. Consider the transformation $Y = X^4$. Then, the Jacobian of the inverse transformation is

(A) $\frac{3}{4}\, y^{-\frac{3}{4}}$

(B) $y^{\frac{3}{4}}$

(C) $\frac{1}{4}\, y^{-\frac{3}{4}}$

(D) $y^{\frac{1}{4}}$

Answer: C

JAM 2026, Q.35 MSQ (one or more correct), 2 marks [5a]

Let X be a random variable with the probability density function

$$f_X(x) = \begin{cases} \frac{1}{2}, & 0 < x < 1, \\ \frac{1}{4}, & 1 < x < 3, \\ 0, & \text{otherwise.} \end{cases}$$

If $Y = X^2$ and $Z = \frac{1}{X}$, then which of the following statements is/are true?

(A) A probability density function of Y is given by

$$f_Y(y) = \begin{cases} \frac{1}{4\sqrt{y}}, & 0 < y < 1, \\ \frac{1}{8\sqrt{y}}, & 1 < y < 9, \\ 0, & \text{otherwise} \end{cases}$$

(B) A probability density function of Z is given by

$$f_Z(z) = \begin{cases} \frac{1}{4z^2}, & \frac{1}{3} < z < 1, \\ \frac{1}{2z^2}, & 1 < z < \infty, \\ 0, & \text{otherwise} \end{cases}$$

(C) $E(Y) < \infty$ and $E(Z) < \infty$

(D) The distribution of Y is positively skewed

Answer: A; B; D

JAM 2026, Q.54 NAT, 2 marks [5a]

Let X be a random variable with the probability density function

$$f(x) = \begin{cases} \frac{3}{4}x(2-x), & 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

If $Z = |X - 1|$ and m is the median of Z, then $12m - 4m^3$ equals __________ (in integer)

Answer: 4 to 4

5.2 5b. Expectation, moments, quantiles, skewness/kurtosis, m.g.f.

JAM 2022, Q.5 MCQ, 1 mark [5b]

If $M(t)$, $t \in \mathbb{R}$, is the moment generating function of a random variable, then which one of the following is NOT the moment generating function of any random variable?

(A) $\frac{5e^{-5t}}{1-4t^2}M(t)$, $|t| < \frac{1}{2}$

(B) $e^{-t}M(t)$, $t \in \mathbb{R}$

(C) $\frac{1+e^t}{2(2-e^t)}M(t)$, $t < \ln 2$

(D) $M(4t)$, $t \in \mathbb{R}$

Answer: A

JAM 2022, Q.21 MCQ, 2 marks [5b]

In a store, the daily demand for milk (in litres) is a random variable having $Exp(\lambda)$ distribution, where $\lambda > 0$. At the beginning of the day, the store purchases $c\ (> 0)$ litres of milk at a fixed price $b\ (> 0)$ per litre. The milk is then sold to the customers at a fixed price $s\ (> b)$ per litre. At the end of the day, the unsold milk is discarded. Then the value of c that maximizes the expected net profit for the store equals

(A) $-\frac{1}{\lambda}\ln\left(\frac{b}{s}\right)$

(B) $-\frac{1}{\lambda}\ln\left(\frac{b}{s+b}\right)$

(C) $-\frac{1}{\lambda}\ln\left(\frac{s-b}{s}\right)$

(D) $-\frac{1}{\lambda}\ln\left(\frac{s}{s+b}\right)$

Answer: A

JAM 2022, Q.46 NAT, 1 mark [5b]

Let X be a random variable denoting the amount of loss in a business. The moment generating function of X is

$$M(t) = \left(\frac{2}{2-t}\right)^2, \quad t < 2.$$

If an insurance policy pays 60% of the loss, then the variance of the amount paid by the insurance policy equals __________ (round off to 2 decimal places)

Answer: 0.17 to 0.19

JAM 2023, Q.9 MCQ, 1 mark [5b]

Let X be a continuous random variable such that $P(X \geq 0) = 1$ and $\mathrm{Var}(X) < \infty$. Then, $E(X^2)$ is

(A) $2\int_0^\infty x^2\, P(X > x)\, dx$

(B) $\int_0^\infty x^2\, P(X > x)\, dx$

(C) $2\int_0^\infty x\, P(X > x)\, dx$

(D) $\int_0^\infty x\, P(X > x)\, dx$

Answer: C

JAM 2023, Q.22 MCQ, 2 marks [5b]

Let X be a random variable such that the moment generating function of X exists in a neighborhood of zero and

$$E(X^n) = (-1)^n \frac{2}{5} + \frac{2^{n+1}}{5} + \frac{1}{5}, \qquad n = 1, 2, 3, \ldots.$$

Then, $P\left(\left|X - \frac{1}{2}\right| > 1\right)$ equals

(A) $\frac{1}{5}$

(B) $\frac{2}{5}$

(C) $\frac{3}{5}$

(D) $\frac{4}{5}$

Answer: D

JAM 2023, Q.39 MSQ (one or more correct), 2 marks [5b]

Let X be a random variable having the probability density function

$$f(x) = \begin{cases} \dfrac{5}{x^6}, & \text{if } x > 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then, which of the following statements is/are TRUE for the distribution of X?

(A) The coefficient of variation is $\frac{4}{\sqrt{15}}$

(B) The first quartile is $\left(\frac{3}{4}\right)^{\frac{1}{5}}$

(C) The median is $(2)^{\frac{1}{5}}$

(D) The upper bound obtained by Chebyshev's inequality for $P\left(X \geq \frac{5}{2}\right)$ is $\frac{1}{15}$

Answer: C; D

JAM 2024, Q.19 MCQ, 2 marks [5b]

Let $f_1(x)$ be the probability density function of the $N(0,1)$ distribution and $f_2(x)$ be the probability density function of the $N(0,6)$ distribution. Let Y be a random variable with probability density function

$$f(x) = 0.6\, f_1(x) + 0.4\, f_2(x), \quad -\infty < x < \infty.$$

Then $\mathrm{Var}(Y)$ is equal to

(A) 7

(B) 3

(C) 3.5

(D) 1

Answer: B

JAM 2024, Q.21 MCQ, 2 marks [5b]

Let X be a random variable such that X and $-X$ have the same distribution. Let $Y = X^2$ be a continuous random variable with the probability density function

$$g(y) = \begin{cases} \dfrac{e^{-\frac{y}{2}}}{\sqrt{2\pi y}}, & \text{if } y > 0 \\ 0, & \text{if } y \leq 0 \end{cases}.$$

Then $E\left((X-1)^4\right)$ is equal to

(A) 9

(B) 10

(C) 11

(D) 12

Answer: B

JAM 2024, Q.35 MSQ (one or more correct), 2 marks [5b]

Let X be a continuous random variable with a probability density function f and the moment generating function $M(t)$. Suppose that $f(x) = f(-x)$ for all $x \in \mathbb{R}$ and the moment generating function $M(t)$ exists for $t \in (-1,1)$. Then which of the following statements is/are correct?

(A) $P(X = -X) = 1$

(B) 0 is the median of X

(C) $M(t) = M(-t)$ for all $t \in (-1, 1)$

(D) $E(X) = 1$

Answer: B; C

JAM 2024, Q.45 NAT, 1 mark [5b]

Let X be a random variable with the moment generating function

$$M(t) = \frac{(1 + 3e^t)^2}{16}, \quad -\infty < t < \infty.$$

Let $\alpha = E(X) - \mathrm{Var}(X)$. Then the value of 8α is equal to __________ (answer in integer)

Answer: 9 to 9

JAM 2024, Q.56 NAT, 2 marks [5b]

Consider a coin for which the probability of obtaining head in a single toss is $\frac{1}{3}$. Sunita tosses the coin once. If head appears, she receives a random amount of X rupees, where X has the Exp $\left(\frac{1}{9}\right)$ distribution. If tail appears, she loses a random amount of Y rupees, where Y has the Exp $\left(\frac{1}{3}\right)$ distribution. Her expected gain (in rupees) is equal to __________ (answer in integer)

Answer: 1 to 1

JAM 2025, Q.6 MCQ, 1 mark [5b]

Suppose X is a Poisson(λ) random variable. Define $Y = (-1)^X X$. Then the expected value of Y is

(A) $-\lambda e^{-2\lambda}$

(B) $-\lambda e^{-\lambda}$

(C) $\lambda e^{-2\lambda}$

(D) $-\lambda$

Answer: A

JAM 2025, Q.18 MCQ, 2 marks [5b]

Consider a circle C with unit radius and center at $A = (0, 0)$. Let $B = (1, 0)$. Suppose $\Theta \sim \mathrm{U}(0, \pi)$ and $D = (\cos\Theta, \sin\Theta)$. Note that the angle $\angle DAB = \Theta$. Then the expected area of the triangle ABD is

(A) $\dfrac{1}{\pi}$

(B) $\dfrac{2}{\pi}$

(C) $\dfrac{1}{2\pi}$

(D) 1

Answer: A

JAM 2025, Q.56 NAT, 2 marks [5b]

Let X be a real valued random variable with $\mathbb{E}(X) = 1$, $\mathbb{E}(X^2) = 4$, $\mathbb{E}(X^4) = 16$. Then $\mathbb{E}(X^3)$ is equal to ________

(answer in integer)

Answer: 4 to 4

JAM 2025, Q.58 NAT, 2 marks [5b]

Let X be a random variable with probability density function

$$f(x) = \begin{cases} 3x^2 & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Let δ denote the conditional expectation of X given that $X \leq \frac{1}{2}$. Then the value of 80δ is equal to ________

(answer in integer)

Answer: 30 to 30

JAM 2026, Q.20 MCQ, 2 marks [5b]

Let X be a random variable having the moment generating function $M_X(t) = e^{2(e^t-1)}$, $t \in \mathbb{R}$. Then, $\text{Var}(2X+3)$ equals

(A) 16

(B) 4

(C) 6

(D) 8

Answer: D

JAM 2026, Q.21 MCQ, 2 marks [5b]

If X is a random variable with the cumulative distribution function

$$F(x) = \begin{cases} 0, & x \leq 0, \\ 1 - e^{-\frac{x^2}{2}}, & x > 0, \end{cases}$$

then $E(X)$ equals

(A) $\sqrt{\frac{\pi}{2}}$

(B) $\sqrt{2\pi}$

(C) $\sqrt{\frac{2}{\pi}}$

(D) 1

Answer: A

5.3 5c. Markov and Chebyshev inequalities

JAM 2022, Q.20 MCQ, 2 marks [5c]

Let X be a random variable with the moment generating function

$$M(t) = \frac{1}{(1-4t)^5}, \quad t < \frac{1}{4}.$$

Then the lower bounds for $P(X < 40)$, using Chebyshev's inequality and Markov's inequality, respectively, are

(A) $\frac{4}{5}$ and $\frac{1}{2}$

(B) $\frac{5}{6}$ and $\frac{1}{2}$

(C) $\frac{4}{5}$ and $\frac{5}{6}$

(D) $\frac{5}{6}$ and $\frac{5}{6}$

Answer: A

JAM 2024, Q.5 MCQ, 1 mark [5c]

Let Y be a continuous random variable such that $P(Y > 0) = 1$ and $E(Y) = 1$. For $p \in (0, 1)$, let ξ_p denote the p^{th} quantile of the probability distribution of the random variable Y. Then which of the following statements is always correct?

(A) $\xi_{0.75} \geq 5$

(B) $\xi_{0.75} \leq 4$

(C) $\xi_{0.25} \geq 4$

(D) $\xi_{0.25} = 2$

Answer: B

JAM 2026, Q.45 NAT, 1 mark [5c]

Let X be a continuous random variable with mean 2 and variance 4. Using Chebyshev's inequality, the lower bound for $P(-4 \leq X \leq 8)$ equals __________ (rounded off to two decimal places)

Answer: 0.88 to 0.90

5.4 5d. Standard discrete distributions

JAM 2022, Q.6 MCQ, 1 mark [5d]

Let X be a random variable having binomial distribution with parameters n (> 1) and p $(0 < p < 1)$. Then $E\left(\frac{1}{1+X}\right)$ equals

(A) $\dfrac{1 - (1-p)^{n+1}}{(n+1)p}$

(B) $\dfrac{1 - p^{n+1}}{(n+1)(1-p)}$

(C) $\dfrac{(1-p)^{n+1}}{n(1-p)}$

(D) $\dfrac{1 - p^{n}}{(n+1)p}$

Answer: A

JAM 2022, Q.19 MCQ, 2 marks [5d]

Suppose that a fair coin is tossed repeatedly and independently. Let X denote the number of tosses required to obtain for the first time a tail that is immediately preceded by a head. Then $E(X)$ and $P(X > 4)$, respectively, are

(A) 4 and $\frac{5}{16}$

(B) 4 and $\frac{11}{16}$

(C) 6 and $\frac{5}{16}$

(D) 6 and $\frac{11}{16}$

Answer: A

JAM 2022, Q.45 NAT, 1 mark [5d]

Consider a sequence of independent Bernoulli trials, where $\frac{3}{4}$ is the probability of success in each trial. Let X be a random variable defined as follows: If the first trial is a success, then X counts the number of failures before the next success. If the first trial is a failure, then X counts the number of successes before the next failure. Then $2E(X)$ equals __________

Answer: 2

JAM 2023, Q.23 MCQ, 2 marks [5d]

Let X be a random variable having a probability mass function $p(x)$ which is positive only for non-negative integers. If

$$p(x+1) = \left(\frac{\ln 3}{x+1}\right) p(x), \qquad x = 0, 1, 2, \ldots,$$

then $\mathrm{Var}(X)$ equals

(A) $\ln 3$

(B) $\ln 6$

(C) $\ln 9$

(D) $\ln 18$

Answer: A

JAM 2024, Q.7 MCQ, 1 mark [5d]

Let X be a random variable having the Poisson distribution with mean 1. Let $g \colon \mathbb{N} \cup \{0\} \to \mathbb{R}$ be defined by

$$g(x) = \begin{cases} 1, & \text{if } x \in \{0, 2\} \\ 0, & \text{if } x \notin \{0, 2\} \end{cases}.$$

Then $E(g(X))$ is equal to

(A) e^{-1}

(B) $2e^{-1}$

(C) $\frac{5}{2}e^{-1}$

(D) $\frac{3}{2}e^{-1}$

Answer: D

JAM 2024, Q.44 NAT, 1 mark [5d]

If 12 fair dice are independently rolled, then the probability of obtaining at least two sixes is equal to __________ (round off to 2 decimal places)

Answer: 0.60 to 0.65

JAM 2025, Q.29 MCQ, 2 marks [5d]

Suppose (X_i, Y_i), $i = 1, 2, \ldots, 200$, are i.i.d. random vectors each having joint probability density function

$$f(x, y) = \begin{cases} \dfrac{1}{25\pi} & \text{if } x^2 + y^2 \leq 25, \\ 0 & \text{otherwise.} \end{cases}$$

Let M be the cardinality of the set $\{i \in \{1, 2, \ldots, 200\} : X_i^2 + Y_i^2 \leq 0.25\}$. Then $\mathbb{P}(M \geq 1)$ is closest to

(A) $\Phi(0.5)$

(B) $1 - e^{-1}$

(C) $\Phi(3)$

(D) $1 - e^{-2}$

Answer: D

5.5 5e. Standard continuous distributions

JAM 2023, Q.46 NAT, 1 mark [5e]

Let X_1, X_2, X_3 be i.i.d. random variables, each having the $N(2, 4)$ distribution. If

$$P(2X_1 - 3X_2 + 6X_3 > 17) = 1 - \Phi(\beta),$$

then β equals ________

Answer: 0.5 to 0.5

JAM 2023, Q.57 NAT, 2 marks [5e]

Let $X_1 \sim \text{Gamma}(1, 4)$, $X_2 \sim \text{Gamma}(2, 2)$ and $X_3 \sim \text{Gamma}(3, 4)$ be three independent random variables. If $Y = X_1 + 2X_2 + X_3$, then $E\left(\left(\frac{Y}{4}\right)^4\right)$ equals ________

Answer: 3024 to 3024

JAM 2023, Q.60 NAT, 2 marks [5e]

Let $X_1 \sim N(2, 1)$, $X_2 \sim N(-1, 4)$ and $X_3 \sim N(0, 1)$ be mutually independent random variables. Then, the probability that exactly two of these three random variables are less than 1, equals ________ (round off to two decimal places)

Answer: 0.63 to 0.65

JAM 2026, Q.40 MSQ (one or more correct), 2 marks [5e]

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n $(n > 1)$ from an $\text{Exp}(\beta)$ distribution, where $\beta > 0$. If $T = \sum_{i=1}^{n} X_i$, then which of the following statements is/are true?

(A) T follows a gamma distribution with mean $(n - 1)\beta$ and variance $(n - 1)\beta^2$

(B) $\frac{2T}{\beta}$ follows a χ^2_{2n} distribution

(C) $\frac{\beta T}{2}$ follows a gamma distribution with mean $\frac{n\beta^2}{2}$ and variance $\frac{n\beta^4}{4}$

(D) The mode of T is $(n - 1)\beta$

Answer: B; C; D

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6 Multivariate Distributions

6.1 6a. Joint, marginal and conditional distributions; independence

JAM 2022, Q.44 NAT, 1 mark [6a]

Let X and Y be two independent and identically distributed random variables having $U(0, 1)$ distribution. Then $P(X^2 < Y < X)$ equals ________ (round off to 2 decimal places)

Answer: 0.15 to 0.18

JAM 2023, Q.17 MCQ, 2 marks [6a]

Let (X, Y) be a random vector having the joint probability density function

$$f(x,y) = \begin{cases} \alpha\,|x|, & \text{if } x^2 \le y \le 2x^2,\ -1 \le x \le 1, \\ 0, & \text{otherwise,} \end{cases}$$

where α is a positive constant. Then, $P(X > Y)$ equals

(A) $\frac{5}{48}$

(B) $\frac{7}{48}$

(C) $\frac{5}{24}$

(D) $\frac{7}{24}$

Answer: B

JAM 2023, Q.21 MCQ, 2 marks [6a]

Let (X, Y, Z) be a random vector having the joint probability density function

$$f(x,y,z) = \begin{cases} \dfrac{1}{2xy}, & \text{if } 0 < z < y < x < 1, \\ \dfrac{1}{2x^2}, & \text{if } 0 < z < x < y < 2x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Then, which one of the following statements is FALSE?

(A) $P(Z < Y < X) = \frac{1}{2}$

(B) $P(X < Y < Z) = 0$

(C) $E(\min\{X, Y\}) = \frac{1}{4}$

(D) $\mathrm{Var}\left(Y \,\middle|\, X = \frac{1}{2}\right) = \frac{1}{12}$

Answer: C

JAM 2023, Q.29 MCQ, 2 marks [6a]

Let X and Y be jointly distributed random variables such that, for every fixed $\lambda > 0$, the conditional distribution of X given $Y = \lambda$ is the Poisson distribution with mean λ. If the distribution of Y is Gamma $\left(2, \frac{1}{2}\right)$, then the value of $P(X = 0) + P(X = 1)$ equals

(A) $\frac{7}{27}$

(B) $\frac{20}{27}$

(C) $\frac{8}{27}$

(D) $\frac{16}{27}$

Answer: B

JAM 2024, Q.37 MSQ (one or more correct), 2 marks [6a]

Let X and Y be continuous random variables having the joint probability density function

$$f(x,y) = \begin{cases} e^{-x}, & \text{if } 0 \le y < x < \infty \\ 0, & \text{otherwise} \end{cases}.$$

Then which of the following statements is/are correct?

(A) $P(Y^2 = 3X) = 0$

(B) $P(X > 2Y) = \frac{1}{2}$

(C) $P(X - Y \geq 1) = e^{-1}$

(D) $P(X > \ln 2 \mid Y > \ln 3) = 0$

Answer: A; B; C

JAM 2025, Q.19 MCQ, 2 marks [6a]

Suppose $Y \sim \mathrm{U}(0,1)$ and the conditional distribution of X given $Y = y$ is $\mathrm{Bin}(6, y)$, for $0 < y < 1$. Then the probability that $(X + 1)$ is an even number is

(A) $\dfrac{3}{7}$

(B) $\dfrac{1}{2}$

(C) $\dfrac{4}{7}$

(D) $\dfrac{5}{14}$

Answer: A

JAM 2025, Q.35 MSQ (one or more correct), 2 marks [6a]

The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} c(x+y) & \text{if } 0 \leq x, y \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

for some constant c. Which of the following statements is/are correct?

(A) $c = 1$

(B) X and Y are independent

(C) The probability density function of X is $g(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$

(D) $X + Y$ has a probability density function

Answer: A; D

JAM 2025, Q.48 NAT, 1 mark [6a]

The joint probability density function of the random vector (X, Y, Z) is given by

$$f(x,y,z) = \begin{cases} \dfrac{1}{xy} & \text{if } 0 < z < y < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Then the value of $\mathbb{P}(X > 5Y)$ is equal to ________
(round off to 2 decimal places)

Answer: 0.20 to 0.20

JAM 2026, Q.22 MCQ, 2 marks [6a]

Let the random variables X and Y denote the time to failure of component A and component B, respectively, of an instrument. The joint probability density function of (X, Y) is given by

$$f(x,y) = \begin{cases} \frac{1}{10}\, e^{-\frac{1}{2}(y+3-x)}, & 5 < x < 10,\ y > x - 3, \\ 0, & \text{otherwise.} \end{cases}$$

The probability, that component B fails before component A, is equal to

(A) $\frac{e\sqrt{e}-1}{e\sqrt{e}}$

(B) $\frac{e\sqrt{e}}{e\sqrt{e}+1}$

(C) $\frac{e\sqrt{e}-1}{e\sqrt{e}+1}$

(D) $\frac{1}{e\sqrt{e}}$

Answer: A

6.2 6b. Distribution of functions of random vectors (transformation, Jacobian)

JAM 2022, Q.22 MCQ, 2 marks [6b]

Let X_1, X_2 and X_3 be three independent and identically distributed random variables having $U(0,1)$ distribution. Then $E\left[\left(\frac{\ln X_1}{\ln X_1X_2X_3}\right)^2\right]$ equals

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

Answer: A

JAM 2023, Q.37 MSQ (one or more correct), 2 marks [6b]

Let X and Y be i.i.d. random variables each having the $N(0,1)$ distribution. Let $U = \frac{X}{Y}$ and $Z = |U|$. Then, which of the following statements is/are TRUE?

(A) U has a Cauchy distribution

(B) $E(Z^p) < \infty$, for some $p \geq 1$

(C) $E(e^{tZ})$ does not exist for all $t \in (-\infty, 0)$

(D) $Z^2 \sim F_{1,1}$

Answer: A; D

JAM 2025, Q.17 MCQ, 2 marks [6b]

Let X_1 and X_2 be i.i.d. $N(0,\sigma^2)$ random variables. Define $Z_1 = X_1 + X_2$ and $Z_2 = X_1 - X_2$. Then which one of the following statements is NOT correct?

(A) Z_1 and Z_2 are independently distributed

(B) Z_1 and Z_2 are identically distributed

(C) $\mathbb{P}\left(\frac{Z_1}{Z_2} < 1\right) = 0.5$

(D) $\frac{Z_1}{Z_2}$ and $Z_1^2 + Z_2^2$ are independently distributed

Answer: C

JAM 2025, Q.49 NAT, 1 mark [6b]

Suppose U_1 and U_2 are i.i.d. $\mathrm{U}(0,1)$ random variables. Further, let X be a $\mathrm{Bin}(2, 0.5)$ random variable that is independent of (U_1, U_2). Then

$$36\,\mathbb{P}(U_1 + U_2 > X)$$

is equal to ________
(answer in integer)

Answer: 18 to 18

6.3 6c. Expectation of functions of random vectors, covariance, correlation, joint m.g.f.

JAM 2022, Q.37 MSQ (one or more correct), 2 marks [6c]

Let (X,Y) be a discrete random vector. Then which of the following statements is/are true?

(A) If X and Y are independent, then X^2 and $|Y|$ are also independent.

(B) If the correlation coefficient between X and Y is 1, then $P(Y = aX + b) = 1$ for some $a, b \in \mathbb{R}$

(C) If X and Y are independent and $E[(XY)^2] = 0$, then $P(X = 0) = 1$ or $P(Y = 0) = 1$

(D) If $\mathrm{Var}(X) = 0$, then X and Y are independent

Answer: A; B; C; D

JAM 2022, Q.47 NAT, 1 mark [6c]

Let (X,Y) be a random vector having the joint moment generating function

$$M(t_1,t_2) = \left(\frac{1}{2}e^{-t_1} + \frac{1}{2}e^{t_1}\right)^2 \left(\frac{1}{2} + \frac{1}{2}e^{t_2}\right)^2, \quad (t_1,t_2) \in \mathbb{R}^2.$$

Then $P(|X+Y| = 2)$ equals __________ (round off to 2 decimal places)

Answer: 0.23 to 0.27

JAM 2023, Q.12 MCQ, 2 marks [6c]

Let X and Y be random variables such that $X \sim N(1,2)$ and $P\left(Y = \frac{X}{2} + 1\right) = 1$. Let $\alpha = \mathrm{Cov}(X,Y)$, $\beta = E(Y)$ and $\gamma = \mathrm{Var}(Y)$. Then, the value of $\alpha + 2\beta + 4\gamma$ equals

(A) 5

(B) 6

(C) 7

(D) 8

Answer: B

JAM 2024, Q.23 MCQ, 2 marks [6c]

Let the random vector (X,Y) have the joint probability density function

$$f(x,y) = \begin{cases} \dfrac{1}{x}, & \text{if } 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}.$$

Then $\mathrm{Cov}(X,Y)$ is equal to

(A) $\frac{1}{6}$

(B) $\frac{1}{12}$

(C) $\frac{1}{18}$

(D) $\frac{1}{24}$

Answer: D

JAM 2024, Q.36 MSQ (one or more correct), 2 marks [6c]

Let X and Y be independent random variables having $\text{Bin}(18, 0.5)$ and $\text{Bin}(20, 0.5)$ distributions, respectively. Further, let $U = \min\{X, Y\}$ and $V = \max\{X, Y\}$. Then which of the following statements is/are correct?

(A) $E(U + V) = 19$

(B) $E(|X - Y|) = E(V - U)$

(C) $\text{Var}(U + V) = 16$

(D) $38 - (X + Y)$ has $\text{Bin}(38, 0.5)$ distribution

Answer: A; B; D

JAM 2024, Q.57 NAT, 2 marks [6c]

Let Θ be a random variable having $U(0, 2\pi)$ distribution. Let $X = \cos\Theta$ and $Y = \sin\Theta$. Let ρ be the correlation coefficient between X and Y. Then 100ρ is equal to __________ (answer in integer)

Answer: 0 to 0

JAM 2025, Q.4 MCQ, 1 mark [6c]

Suppose $Z_1, Z_2, \ldots, Z_{128}$ are i.i.d. $\text{Bin}(1, 0.5)$ random variables. Define

$$\mathbf{X} = (Z_1, Z_2, \ldots, Z_{64})^T \quad \text{and} \quad \mathbf{Y} = (Z_{65}, Z_{66}, \ldots, Z_{128})^T.$$

Then the value of $\text{Var}(\mathbf{X}^T\mathbf{Y})$ is

(A) 4

(B) 8

(C) 12

(D) 16

Answer: C

JAM 2025, Q.34 MSQ (one or more correct), 2 marks [6c]

The joint moment generating function of (X, Y) is given by

$$M_{X,Y}(s,t) = \left(\frac{1}{4} + \frac{1}{2}e^s + \frac{1}{4}e^t\right)^2, \quad (s,t) \in \mathbb{R}^2.$$

Then which of the following statements is/are correct?

(A) $\mathbb{E}(X) = 1$

(B) $\mathbb{E}(Y^2) = \dfrac{3}{8}$

(C) $\text{Cov}(X, Y) = -\dfrac{1}{4}$

(D) $\text{Var}(X) = \dfrac{1}{2}$

Answer: A; C; D

JAM 2026, Q.6 MCQ, 1 mark [6c]

Let (X, Y) be a random vector with $\text{Var}(X) = \text{Var}(Y) = 1$ and $\text{Cov}(X, Y) = -0.5$. For which one of the following values of b, the random variables $U = bX + Y$ and $V = X + bY$ are uncorrelated?

(A) $2-\sqrt{3}$

(B) $\sqrt{3}-2$

(C) $\sqrt{3}$

(D) 2

Answer: A

6.4 6d. Conditional expectation and conditional variance

JAM 2022, Q.7 MCQ, 1 mark [6d]

Let (X, Y) be a random vector having the joint probability density function

$$f(x,y) = \begin{cases} \dfrac{\sqrt{2}}{\sqrt{\pi}}\, e^{-2x}\, e^{-\frac{(y-x)^2}{2}}, & 0 < x < \infty,\ -\infty < y < \infty \\ 0, & \text{otherwise.} \end{cases}$$

Then $E(Y)$ equals

(A) $\frac{1}{2}$

(B) 2

(C) 1

(D) $\frac{1}{4}$

Answer: A

JAM 2023, Q.6 MCQ, 1 mark [6d]

Let X_1, X_2, X_3 be a random sample from an $N(\theta, 1)$ distribution, where $\theta \in \mathbb{R}$ is an unknown parameter. Then, which one of the following conditional expectations does NOT depend on θ?

(A) $E(X_1 + X_2 - X_3 \mid X_1 + X_2)$

(B) $E(X_1 + X_2 - X_3 \mid X_2 + X_3)$

(C) $E(X_1 + X_2 - X_3 \mid X_1 - X_3)$

(D) $E(X_1 + X_2 - X_3 \mid X_1 + X_2 + X_3)$

Answer: D

JAM 2023, Q.56 NAT, 2 marks [6d]

Let $X_1, X_2, \ldots, X_5$ be i.i.d. random variables, each having the $\text{Bin}\left(1, \frac{1}{2}\right)$ distribution. Let $K = X_1 + X_2 + \cdots + X_5$ and

$$U = \begin{cases} 0, & \text{if } K = 0, \\ X_1 + X_2 + \cdots + X_K, & \text{if } K = 1, 2, \ldots, 5. \end{cases}$$

Then, $E(U)$ equals __________

Answer: 1.5 to 1.5

JAM 2024, Q.22 MCQ, 2 marks [6d]

Suppose that random variable X has $\text{Exp}\left(\frac{1}{5}\right)$ distribution and, for any $x > 0$, the conditional distribution of random variable Y, given $X = x$, is $N(x, 2)$. Then $\text{Var}(X + Y)$ is equal to

(A) 52

(B) 50

(C) 2

(D) 102

Answer: D

JAM 2025, Q.55 NAT, 2 marks [6d]

Consider a sequence of independent Bernoulli trials with success probability $p = \frac{1}{7}$. Then the expected number of trials required to get two consecutive successes for the first time is equal to ________ (answer in integer)

Answer: 56 to 56

JAM 2026, Q.36 MSQ (one or more correct), 2 marks [6d]

Let (X, Y) be a random vector with the joint probability density function

$$f(x,y) = \begin{cases} x\, e^{-x(y+1)}, & x > 0,\ y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Which of the following statements is/are true?

(A) The marginal distribution of X is Exp(1)

(B) $E(Y|X = 4) = 0.25$

(C) $E(XY) = 1.5$

(D) $E(Y)$ is not finite

Answer: A; B; D

JAM 2026, Q.55 NAT, 2 marks [6d]

If (X, Y) is a random vector with the joint probability mass function

$$f(x,y) = \begin{cases} 2e^{-3}\, \frac{3^{2x-y-1}}{x!}, & x = 0, 1, 2, \ldots;\ y = x, x+1, x+2, \ldots, \\ 0, & \text{otherwise,} \end{cases}$$

then $\mathrm{Var}(Y|X = 2)$ equals __________ (rounded off to two decimal places)

Answer: 0.74 to 0.76

6.5 6e. Multinomial distribution

JAM 2022, Q.56 NAT, 2 marks [6e]

A vaccine, when it is administered to an individual, produces no side effects with probability $\frac{4}{5}$, mild side effects with probability $\frac{2}{15}$ and severe side effects with probability $\frac{1}{15}$. Assume that the development of side effects is independent across individuals. The vaccine was administered to 1000 randomly selected individuals. If X_1 denotes the number of individuals who developed mild side effects and X_2 denotes the number of individuals who developed severe side effects, then the coefficient of variation of $X_1 + X_2$ equals __________ (round off to 2 decimal places)

Answer: 0.05 to 0.07

JAM 2023, Q.34 MSQ (one or more correct), 2 marks [6e]

Let (X_1, X_2) be a random vector having the probability mass function

$$f(x_1,x_2) = \begin{cases} \dfrac{c}{x_1!\, x_2!\, (12 - x_1 - x_2)!}, & \text{if } x_1, x_2 \in \{0, 1, \ldots, 12\},\ x_1 + x_2 \le 12, \\ 0, & \text{otherwise,} \end{cases}$$

where c is a real constant. Then, which of the following statements is/are TRUE?

(A) $E(X_1 + X_2) = 8$

(B) $\mathrm{Var}(X_1 + X_2) = \frac{8}{3}$

(C) $\mathrm{Cov}(X_1, X_2) = -\frac{5}{3}$

(D) $\mathrm{Var}(X_1 + 2X_2) = 8$

Answer: A; B; D

6.6 6f. Bivariate normal distribution

JAM 2022, Q.23 MCQ, 2 marks [6f]

Let (X, Y) be a random vector having bivariate normal distribution with parameters $E(X) = 0$, $\mathrm{Var}(X) = 1$, $E(Y) = -1$, $\mathrm{Var}(Y) = 4$ and $\rho(X, Y) = -\frac{1}{2}$, where $\rho(X, Y)$ denotes the correlation coefficient between X and Y. Then $P(X + Y > 1 \mid 2X - Y = 1)$ equals

(A) $\Phi\left(-\frac{1}{2}\right)$

(B) $\Phi\left(-\frac{1}{3}\right)$

(C) $\Phi\left(-\frac{1}{4}\right)$

(D) $\Phi\left(-\frac{4}{3}\right)$

Answer: D

JAM 2022, Q.36 MSQ (one or more correct), 2 marks [6f]

Let X and Y be two independent random variables having $N(0, \sigma_1^2)$ and $N(0, \sigma_2^2)$ distributions, respectively, where $0 < \sigma_1 < \sigma_2$. Then which of the following statements is/are true?

(A) $X + Y$ and $X - Y$ are independent

(B) $2X + Y$ and $X - Y$ are independent if $2\sigma_1^2 = \sigma_2^2$

(C) $X + Y$ and $X - Y$ are identically distributed

(D) $X + Y$ and $2X - Y$ are independent if $2\sigma_1^2 = \sigma_2^2$

Answer: B; C; D

JAM 2023, Q.52 NAT, 2 marks [6f]

Let the random vector (X_1, X_2) follow the bivariate normal distribution with

$$E(X_1) = E(X_2) = 1, \qquad \mathrm{Var}(X_2) = 4\,\mathrm{Var}(X_1) = 4 \quad \text{and} \quad \mathrm{Cov}(X_1, X_2) = 1.$$

Then, $\mathrm{Var}\left(X_1 + X_2 \,\middle|\, X_1 = \frac{1}{2}\right)$ equals __________

Answer: 3 to 3

JAM 2024, Q.24 MCQ, 2 marks [6f]

Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_{20}, Y_{20})$ be a random sample from the $N_2\left(0, 0, 1, 1, \frac{3}{4}\right)$ distribution. Define $\bar{X} = \frac{1}{20}\sum_{i=1}^{20} X_i$ and $\bar{Y} = \frac{1}{20}\sum_{i=1}^{20} Y_i$. Then $\mathrm{Var}(\bar{X} - \bar{Y})$ is equal to

(A) $\frac{1}{16}$

(B) $\frac{1}{40}$

(C) $\frac{1}{10}$

(D) $\frac{3}{40}$

Answer: B

JAM 2025, Q.23 MCQ, 2 marks [6f]

Let (X, Y) have the $N_2(0, 0, 1, 1, 0.25)$ distribution. Then the correlation coefficient between e^X and e^{2Y} is

(A) $\dfrac{e^3 - e^{5/2}}{\left(e^5(e-1)(e^4-1)\right)^{1/2}}$

(B) $\dfrac{e^3 - e^{5/2}}{\left(e^4(e-1)(e^5-1)\right)^{1/2}}$

(C) $\dfrac{e^2 - e^{5/2}}{\left(e^5(e-1)(e^4-1)\right)^{1/2}}$

(D) $\dfrac{e^{3/2} - e^{5/2}}{\left(e^5(e^2-1)(e^4-1)\right)^{1/2}}$

Answer: A

JAM 2025, Q.25 MCQ, 2 marks [6f]

If $(X, Y) \sim N_2(0, 0, 1, 1, 0.5)$, then the value of $\mathbb{E}\left(e^{-XY}\right)$ is

(A) $\dfrac{2}{\sqrt{5}}$

(B) $\dfrac{2}{\sqrt{3}}$

(C) $\dfrac{1}{\sqrt{2}}$

(D) $\dfrac{1}{2}$

Answer: A

JAM 2025, Q.60 NAT, 2 marks [6f]

Suppose (X, Y) has the $N_2(3, 0, 4, 1, 0.5)$ distribution. Then $4\,\mathrm{Cov}(X + Y, Y^3)$ is equal to ________ (answer in integer)

Answer: 24 to 24

JAM 2026, Q.46 NAT, 1 mark [6f]

Let (X, Y) be a random vector having the bivariate normal distribution with $E(X) = 2$, $E(Y) = 10$, $\mathrm{Var}(X) = 9$, $\mathrm{Var}(Y) = 25$, and the correlation coefficient between X and Y equals $\rho > 0$. If $P(4 < Y < 16 \mid X = 2) = 0.8664$, then ρ equals __________ (rounded off to two decimal places)
[You may use $\Phi(1.5) = 0.9332$, $\Phi(2) = 0.9772$]

Answer: 0.59 to 0.61

7 Limit Theorems

7.1 7a. Modes of convergence and their inter-relations

JAM 2022, Q.57 NAT, 2 marks [7a]

Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables having $U(0, 1)$ distribution. Let $Y_n = n \min\{X_1, X_2, \ldots, X_n\}$, $n \geq 1$. If Y_n converges to Y in distribution, then the median of Y equals __________ (round off to 2 decimal places)

Answer: 0.67 to 0.71

JAM 2023, Q.5 MCQ, 1 mark [7a]

Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables each having $U(0,1)$ distribution. Let Y be a random variable having the distribution function G. Suppose that

$$\lim_{n\to\infty} P\left(\frac{X_1 + X_2 + X_n}{4} \leq x\right) = G(x), \quad \text{for all } x \in \mathbb{R}.$$

Then, $\operatorname{Var}(Y)$ equals

(A) $\frac{1}{12}$

(B) $\frac{1}{32}$

(C) $\frac{1}{48}$

(D) $\frac{1}{64}$

Answer: D

JAM 2025, Q.30 MCQ, 2 marks [7a]

Let $\{X_n\}_{n\geq 1}$ be a sequence of random variables, where $X_n \sim \operatorname{Bin}(n, p_n)$ with $p_n \in (0,1)$. Which of the following conditions implies that $X_n \xrightarrow{d} 0$ as $n \to \infty$?

(A) $\lim_{n\to\infty} p_n = 0$

(B) $\lim_{n\to\infty} \mathbb{P}(X_n = k) = 0$ for each $k \in \mathbb{N}$

(C) $\lim_{n\to\infty} \mathbb{E}(X_n) = 0$

(D) $\sup_{n\geq 1} \operatorname{Var}(X_n) < \infty$

Answer: C

JAM 2026, Q.23 MCQ, 2 marks [7a]

Let $X_1, X_2, \ldots$ be a sequence of independent random variables with

$$P(X_n = n) = \frac{1}{n^2} = 1 - P(X_n = 0), \qquad \text{for } n = 1, 2, \ldots.$$

Consider the following statements.
$\boldsymbol{P}$: $\{X_n\}$ converges in mean-square to 0.
$\boldsymbol{Q}$: $\{X_n\}$ converges in probability to 0.
Then

(A) $\boldsymbol{P}$ is correct and $\boldsymbol{Q}$ is NOT correct

(B) $\boldsymbol{P}$ is NOT correct and $\boldsymbol{Q}$ is correct

(C) Both $\boldsymbol{P}$ and $\boldsymbol{Q}$ are correct

(D) Neither $\boldsymbol{P}$ nor $\boldsymbol{Q}$ is correct

Answer: B

7.2 7b. WLLN, SLLN, Central Limit Theorem

JAM 2022, Q.24 MCQ, 2 marks [7b]

Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables having the common probability density function

$$f(x) = \begin{cases} \dfrac{2}{x^3}, & x \geq 1 \\ 0, & \text{otherwise.} \end{cases}$$

If $\lim_{n\to\infty} P\left(\left|\frac{1}{n}\sum_{i=1}^{n} X_i - \theta\right| < \epsilon\right) = 1$ for all $\epsilon > 0$, then θ equals

(A) 4

(B) 2

(C) $\ln 4$

(D) $\ln 2$

Answer: B

JAM 2023, Q.14 MCQ, 2 marks [7b]

Let $X_1, X_2, \ldots$ be a sequence of independent random variables such that

$$P(X_i = i) = \frac{1}{4} \quad \text{and} \quad P(X_i = 2i) = \frac{3}{4}, \qquad i = 1, 2, \ldots.$$

For some real constants c_1 and c_2, suppose that

$$\frac{c_1}{\sqrt{n}} \sum_{i=1}^{n} \frac{X_i}{i} + c_2\sqrt{n} \xrightarrow{d} Z \sim N(0,1), \qquad \text{as } n \to \infty.$$

Then, the value of $\sqrt{3}\,(3c_1 + c_2)$ equals

(A) 2

(B) 3

(C) 4

(D) 5

Answer: D

JAM 2023, Q.15 MCQ, 2 marks [7b]

Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables such that

$$P(X_1 = 0) = P(X_1 = 1) = P(X_1 = 2) = \frac{1}{3}.$$

Let $S_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $T_n = \frac{1}{n}\sum_{i=1}^{n} X_i^2$, $n = 1, 2, \ldots$. Suppose that

$$\alpha_1 = \lim_{n\to\infty} P\left(\left|S_n - \frac{1}{2}\right| < \frac{3}{4}\right), \qquad \alpha_2 = \lim_{n\to\infty} P\left(\left|S_n - \frac{1}{3}\right| < 1\right),$$

$$\alpha_3 = \lim_{n\to\infty} P\left(\left|T_n - \frac{1}{3}\right| < \frac{3}{2}\right) \quad \text{and} \quad \alpha_4 = \lim_{n\to\infty} P\left(\left|T_n - \frac{2}{3}\right| < \frac{1}{2}\right).$$

Then, the value of $\alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4$ equals

(A) 6

(B) 5

(C) 4

(D) 3

Answer: A

JAM 2023, Q.42 NAT, 1 mark [7b]

Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables, each having the probability density function

$$f(x) = \begin{cases} \dfrac{x^2 e^{-x}}{2}, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

For some real constants β, γ and k, suppose that

$$\lim_{n\to\infty} P\left(\frac{1}{n}\sum_{i=1}^{n} X_i \leq x\right) = \begin{cases} 0, & \text{if } x < \beta, \\ kx, & \text{if } \beta \leq x \leq \gamma, \\ k\gamma, & \text{if } x > \gamma. \end{cases}$$

Then, the value of $2\beta + 3\gamma + 6k$ equals __________

Answer: 17 to 17

JAM 2024, Q.25 MCQ, 2 marks [7b]

For $n \in \mathbb{N}$, let X_n be a random variable having the Bin $\left(n, \frac{1}{4}\right)$ distribution. Then

$$\lim_{n\to\infty} \left[P\left(X_n \leq \frac{2n - \sqrt{3n}}{8}\right) + P\left(\frac{n}{6} \leq X_n \leq \frac{n}{3}\right)\right]$$

is equal to

(You may use $\Phi(0.5) = 0.6915$, $\Phi(1) = 0.8413$, $\Phi(1.5) = 0.9332$, $\Phi(2) = 0.9772$)

(A) 1.6915

(B) 1.3085

(C) 1.1587

(D) 0.6915

Answer: B

JAM 2024, Q.38 MSQ (one or more correct), 2 marks [7b]

For $n \geq 2$, let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with $E(X_1) = 0$, $\mathrm{Var}(X_1) = 1$ and $E(X_1^4) < \infty$. Let

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2.$$

Then which of the following statements is/are always correct?

(A) $E(S_n^2) = 1$ for all $n \geq 2$

(B) $\sqrt{n}\,\bar{X}_n \xrightarrow{d} Z$ as $n \to \infty$, where Z has the $N(0,1)$ distribution

(C) $\bar{X}_n$ and S_n^2 are independently distributed for all $n \geq 2$

(D) $\frac{1}{n}\sum_{i=1}^{n} X_i^2 \xrightarrow{P} 2$, as $n \to \infty$

Answer: A; B

JAM 2024, Q.46 NAT, 1 mark [7b]

For $n \in \mathbb{N}$, let $X_1, X_2, \ldots, X_n$ be a random sample from the Cauchy distribution having probability density function

$$f(x) = \frac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty.$$

Let $g\colon \mathbb{R} \to \mathbb{R}$ be defined by

$$g(x) = \begin{cases} x, & \text{if } -1000 \le x \le 1000 \\ 0, & \text{otherwise} \end{cases}.$$

Let

$$\alpha = \lim_{n\to\infty} P\left(\frac{1}{n^{\frac{3}{4}}}\sum_{i=1}^{n} g(X_i) > \frac{1}{2}\right).$$

Then 100α is equal to __________ (answer in integer)

Answer: 0 to 0

JAM 2024, Q.47 NAT, 1 mark [7b]

For $n \in \mathbb{N}$, let $X_1, X_2, \ldots, X_n$ be a random sample from the $F_{20,40}$ distribution. Then, as $n \to \infty$, $\frac{1}{n}\sum_{i=1}^{n}\frac{1}{X_i}$ converges in probability to __________
(round off to 2 decimal places)

Answer: 1.00 to 1.20

JAM 2025, Q.7 MCQ, 1 mark [7b]

Let $\{Y_k\}_{k\ge1}$ be a sequence of i.i.d. $\text{Bin}(1,p)$ random variables, where $0 < p < 1$ is an unknown parameter. Let $\hat{p}_n$ be the maximum likelihood estimator of p based on $Y_1, Y_2, \ldots, Y_n$. It is claimed that:

(I) $\dfrac{\hat{p}_n - p}{\sqrt{\dfrac{p(1-p)}{n}}} \xrightarrow{d} N(0,1)$ as $n \to \infty$

(II) $\dfrac{\hat{p}_n - p}{\sqrt{\dfrac{\hat{p}_n(1-\hat{p}_n)}{n}}} \xrightarrow{d} N(0,1)$ as $n \to \infty$

Which of the following statements is correct?

(A) (I) is correct and (II) is incorrect

(B) (I) is incorrect and (II) is correct

(C) Both (I) and (II) are correct

(D) Both (I) and (II) are incorrect

Answer: C

JAM 2025, Q.24 MCQ, 2 marks [7b]

Let $\{X_k\}_{k\ge1}$ be a sequence of i.i.d. $\text{U}(-1,1)$ random variables. Suppose

$$Y_n = \sqrt{3n}\,\frac{\sum_{i=1}^{n} X_i}{\sum_{i=1}^{n} X_i^4}.$$

Then $\{Y_n\}_{n\ge1}$ converges in distribution as $n \to \infty$ to a

(A) $N(0, 1)$ random variable

(B) random variable degenerate at 0

(C) $N(0, 25)$ random variable

(D) $N(0, 0.04)$ random variable

Answer: C

JAM 2026, Q.47 NAT, 1 mark [7b]

Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables having $U(0, 3)$ distribution. If $Y_n = \frac{1}{n}\sum_{i=1}^{n} X_i^2$, $n \geq 1$, then $\{Y_n\}_{n\geq 1}$ converges in probability to __________ (in integer)

Answer: 3 to 3

JAM 2026, Q.56 NAT, 2 marks [7b]

Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with mean 4 and variance 9. If $T_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, then

$$\lim_{n\to\infty} P\left(4\sqrt{n} - 3 < \sqrt{n}\,T_n < 6 + 4\sqrt{n}\right)$$

equals __________ (rounded off to two decimal places)
[You may use $\Phi(2) = 0.9772$, $\Phi(1.5) = 0.9332$, $\Phi(1) = 0.8413$]

Answer: 0.81 to 0.83

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8 Sampling Distributions

8.1 8a. Order statistics

JAM 2022, Q.8 MCQ, 1 mark [8a]

Let X_1 and X_2 be two independent and identically distributed discrete random variables having the probability mass function

$$f(x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1, 2, 3, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(\min\{X_1, X_2\} \geq 5)$ equals

(A) $\frac{1}{256}$

(B) $\frac{1}{512}$

(C) $\frac{1}{64}$

(D) $\frac{9}{256}$

Answer: A

JAM 2022, Q.27 MCQ, 2 marks [8a]

Let $X_{(1)} < X_{(2)} < \cdots < X_{(9)}$ be the order statistics corresponding to a random sample of size 9 from $U(0, 1)$ distribution. Then which one of the following statements is NOT true?

(A) $E\left(\dfrac{X_{(9)}}{1 - X_{(9)}}\right)$ is finite

(B) $E\left(X_{(5)}\right) = 0.5$

(C) The median of $X_{(5)}$ is 0.5

(D) The mode of $X_{(5)}$ is 0.5

Answer: A

JAM 2022, Q.58 NAT, 2 marks [8a]

Let $X_{(1)} < X_{(2)} < X_{(3)} < X_{(4)} < X_{(5)}$ be the order statistics based on a random sample of size 5 from a continuous distribution with the probability density function

$$f(x) = \begin{cases} \dfrac{1}{x^2}, & 1 < x < \infty \\ 0, & \text{otherwise.} \end{cases}$$

Then the sum of all possible values of $r \in \{1, 2, 3, 4, 5\}$ for which $E(X_{(r)})$ is finite equals __________

Answer: 10

JAM 2023, Q.11 MCQ, 2 marks [8a]

Let X_1 and X_2 be i.i.d. random variables having the common probability density function

$$f(x) = \begin{cases} e^{-x}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

Define $X_{(1)} = \min\{X_1, X_2\}$ and $X_{(2)} = \max\{X_1, X_2\}$. Then, which one of the following statements is FALSE?

(A) $\dfrac{2X_{(1)}}{X_{(2)} - X_{(1)}} \sim F_{2,2}$

(B) $2\left(X_{(2)} - X_{(1)}\right) \sim \chi_2^2$

(C) $E\left(X_{(1)}\right) = \frac{1}{2}$

(D) $P\left(3X_{(1)} < X_{(2)}\right) = \frac{1}{3}$

Answer: D

JAM 2024, Q.8 MCQ, 1 mark [8a]

For $n \in \mathbb{N}$, let Z_n be the smallest order statistic based on a random sample of size n from the $U(0,1)$ distribution. Let $nZ_n \xrightarrow{d} Z$, as $n \to \infty$, for some random variable Z. Then $P(Z \leq \ln 3)$ is equal to

(A) $\frac{1}{4}$

(B) $\frac{2}{3}$

(C) $\frac{3}{4}$

(D) $\frac{1}{3}$

Answer: B

JAM 2024, Q.48 NAT, 1 mark [8a]

Let $X_1, X_2, \ldots, X_{10}$ be a random sample from the Exp(1) distribution. Define

$$W = \max\{e^{-X_1}, e^{-X_2}, \ldots, e^{-X_{10}}\}.$$

Then the value of $22E(W)$ is equal to __________ (answer in integer)

Answer: 20 to 20

JAM 2024, Q.49 NAT, 1 mark [8a]

Let X_1, X_2, X_3 be i.i.d. random variables from a continuous distribution having probability density function

$$f(x) = \begin{cases} \dfrac{1}{2x^3}, & \text{if } x > \frac{1}{2} \\ 0, & \text{if } x \leq \frac{1}{2} \end{cases}.$$

Let $X_{(1)} = \min\{X_1, X_2, X_3\}$. Then the value of $10E\left(X_{(1)}\right)$ is equal to __________ (answer in integer)

Answer: 6 to 6

JAM 2025, Q.47 NAT, 1 mark [8a]

Let X and Y be i.i.d. random variables with probability density function

$$f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x \geq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose $Z = \min\{X, Y\}$, then $\mathbb{E}(Z)$ is equal to ________
(answer in integer)

Answer: 2 to 2

JAM 2025, Q.59 NAT, 2 marks [8a]

Let $X_1, X_2, \ldots, X_7$ be i.i.d. continuous random variables with median θ. If $X_{(1)} < X_{(2)} < \cdots < X_{(7)}$ are the corresponding order statistics, then $\mathbb{P}\left(X_{(2)} > \theta\right)$ is equal to ________
(round off to 3 decimal places).

Answer: 0.062 to 0.063

JAM 2026, Q.50 NAT, 1 mark [8a]

Let X_1, X_2, X_3 be a random sample of size 3 from $U(1, 2)$ distribution. Then $P(X_{(3)} > 1.5)$ equals __________ (rounded off to two decimal places)

Answer: 0.87 to 0.89

JAM 2026, Q.60 NAT, 2 marks [8a]

Let X_1, X_2, X_3 be a random sample of size 3 from Exp(1) distribution. Then, $E\left(4X_{(2)}e^{X_{(2)}}\right)$ equals __________ (in integer)

Answer: 18 to 18

8.2 8b. Chi-square, t and F distributions; sampling distributions of statistics

JAM 2022, Q.26 MCQ, 2 marks [8b]

For $n = 1, 2, 3, \ldots$, let the joint moment generating function of (X, Y_n) be

$$M_{X,Y_n}(t_1, t_2) = e^{\frac{t_1^2}{2}}(1 - 2t_2)^{-\frac{n}{2}}, \quad t_1 \in \mathbb{R},\ t_2 < \frac{1}{2}.$$

If $T_n = \dfrac{\sqrt{n}\,X}{\sqrt{Y_n}}$, $n \geq 1$, then which one of the following statements is true?

(A) The minimum value of n for which $\mathrm{Var}(T_n)$ is finite is 2

(B) $E(T_{10}^3) = 10$

(C) $\mathrm{Var}(X + Y_4) = 7$

(D) $\lim\limits_{n\to\infty} P(|T_n| > 3) = 1 - \dfrac{\sqrt{2}}{\sqrt{\pi}} \displaystyle\int_0^3 e^{-\frac{t^2}{2}}\, dt$

Answer: D

JAM 2022, Q.38 MSQ (one or more correct), 2 marks [8b]

Let X_1, X_2 and X_3 be three independent and identically distributed random variables having $N(0, 1)$ distribution. If

$$U = \frac{2X_1^2}{(X_2 + X_3)^2} \qquad \text{and} \qquad V = \frac{2(X_2 - X_3)^2}{2X_1^2 + (X_2 + X_3)^2},$$

then which of the following statements is/are true?

(A) U has $F_{1,1}$ distribution and V has $F_{1,2}$ distribution

(B) U has $F_{1,1}$ distribution and V has $F_{2,1}$ distribution

(C) U and V are independent

(D) $\frac{1}{2}V(1+U)$ has $F_{2,3}$ distribution

Answer: A; C

JAM 2022, Q.48 NAT, 1 mark [8b]

Let X_1 and X_2 be two independent and identically distributed random variables having χ^2_2 distribution and $W = X_1 + X_2$. Then $P(W > E(W))$ equals __________ (round off to 2 decimal places)

Answer: 0.39 to 0.43

JAM 2023, Q.3 MCQ, 1 mark [8b]

Let $X \sim F_{6,2}$ and $Y \sim F_{2,6}$. If $P(X \leq 2) = \frac{216}{343}$ and $P\left(Y \leq \frac{1}{2}\right) = \alpha$, then 686α equals

(A) 246

(B) 254

(C) 260

(D) 264

Answer: B

JAM 2023, Q.4 MCQ, 1 mark [8b]

Let $Y \sim F_{4,2}$. Then, $P(Y \leq 2)$ equals

(A) 0.60

(B) 0.62

(C) 0.64

(D) 0.66

Answer: C

JAM 2023, Q.32 MSQ (one or more correct), 2 marks [8b]

Let X_1, X_2, X_3 be i.i.d. random variables, each having the $N(0,1)$ distribution. Then, which of the following statements is/are TRUE?

(A) $\dfrac{\sqrt{2}\,(X_1 - X_2)}{\sqrt{(X_1 + X_2)^2 + 2X_3^2}} \sim t_1$

(B) $\dfrac{(X_1 + X_2)^2}{(X_1 - X_2)^2 + 2X_3^2} \sim F_{1,2}$

(C) $E\left(\dfrac{X_1}{X_2^2 + X_3^2}\right) = 0$

(D) $P(X_1 < X_2 + X_3) = \frac{1}{3}$

Answer: C

JAM 2024, Q.9 MCQ, 1 mark [8b]

Let $X_1, X_2, \ldots, X_{20}$ be a random sample from the $N(5,2)$ distribution and let $Y_i = X_{2i} - X_{2i-1}$, $i = 1, 2, \ldots, 10$. Then $W = \frac{1}{4}\sum_{i=1}^{10} Y_i^2$ has the

(A) t_{20} distribution

(B) χ^2_{20} distribution

(C) χ^2_{10} distribution

(D) $N(250, 20)$ distribution

Answer: C

JAM 2024, Q.26 MCQ, 2 marks [8b]

Let $X_1, X_2, \ldots, X_{10}$ be a random sample from the $N(3,4)$ distribution and let $Y_1, Y_2, \ldots, Y_{15}$ be a random sample from the $N(-3,6)$ distribution. Assume that the two samples are drawn independently. Define

$$\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i, \quad \bar{Y} = \frac{1}{15}\sum_{j=1}^{15} Y_j, \quad \text{and} \quad S = \sqrt{\frac{1}{9}\sum_{i=1}^{10}(X_i - \bar{X})^2}.$$

Then the distribution of $U = \frac{\sqrt{5}(\bar{X}+\bar{Y})}{S}$ is

(A) $N\left(0, \frac{4}{5}\right)$

(B) χ^2_9

(C) t_9

(D) t_{23}

Answer: C

JAM 2024, Q.39 MSQ (one or more correct), 2 marks [8b]

Let $X_1, X_2, \ldots, X_{50}$ be a random sample from a $N(0, \sigma^2)$ distribution, where $\sigma > 0$. Define

$$\bar{X}_e = \frac{1}{25}\sum_{i=1}^{25} X_{2i}, \qquad \bar{X}_o = \frac{1}{25}\sum_{i=1}^{25} X_{2i-1},$$

$$S_e = \sqrt{\frac{1}{24}\sum_{i=1}^{25}(X_{2i} - \bar{X}_e)^2} \quad \text{and} \quad S_o = \sqrt{\frac{1}{24}\sum_{i=1}^{25}(X_{2i-1} - \bar{X}_o)^2}.$$

Then which of the following statements is/are correct?

(A) $\frac{5\bar{X}_e}{S_e}$ has t_{24} distribution

(B) $\frac{5(\bar{X}_e+\bar{X}_o)}{\sqrt{S_e^2+S_o^2}}$ has t_{49} distribution

(C) $\frac{49S_o^2}{\sigma^2}$ has χ^2_{49} distribution

(D) $\frac{S_o^2}{S_e^2}$ has $F_{24,24}$ distribution

Answer: A; D

JAM 2025, Q.9 MCQ, 1 mark [8b]

Suppose $X \sim N(0,4)$ and $Y \sim N(0,9)$ are independent random variables. Then the value of $\mathbb{P}(9X^2 + 4Y^2 < 6)$ is

(A) $1 - e^{-1/4}$

(B) $1 - e^{-1/12}$

(C) $1 - e^{-1/6}$

(D) $1 - e^{-1/9}$

Answer: B

JAM 2025, Q.21 MCQ, 2 marks [8b]

Let $X_1, X_2, \ldots, X_5$ be i.i.d. $N(0, \sigma^2)$ random variables. Suppose c is such that

$$\mathbb{E}\left(c\sqrt{\sum_{i=1}^{5} X_i^2}\right) = \sigma.$$

Then the value of c is

(A) $\sqrt{\dfrac{9\pi}{128}}$

(B) $\sqrt{\dfrac{9\pi}{64}}$

(C) $\sqrt{\dfrac{9}{128\pi}}$

(D) $\sqrt{\dfrac{3}{64\pi}}$

Answer: A

JAM 2025, Q.28 MCQ, 2 marks [8b]

Let $X_1, X_2, \ldots, X_n$ be i.i.d. $N(0, 1)$ random variables, where $n > 3$. If

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2,$$

then $\mathrm{Var}\left(\dfrac{\bar{X}}{S}\right)$ is equal to

(A) $\dfrac{(n-3)}{n(n-1)}$

(B) $\dfrac{(n-1)}{n(n-3)}$

(C) $\dfrac{(n-1)}{n(n-2)}$

(D) $\dfrac{(n-2)}{n(n-1)}$

Answer: B

JAM 2025, Q.36 MSQ (one or more correct), 2 marks [8b]

Let $X_1, X_2, \ldots, X_{20}$ be a random sample from a $N(\mu, \sigma^2)$ population. Suppose

$$P = \frac{1}{10}\sum_{i=1}^{10} X_i \quad \text{and} \quad Q = \frac{1}{9}\sum_{i=1}^{10}(X_i - P)^2.$$

Then which of the following statements is/are correct?

(A) $\dfrac{X_{11} + P - X_{12} - X_{20}}{\sqrt{Q}} \sim \sqrt{\dfrac{31}{10}}\, t_9$

(B) $\dfrac{P - X_{15}}{\sqrt{9Q + (X_{18} - \mu)^2}} \sim \dfrac{\sqrt{11}}{10}\, t_{10}$

(C) $\dfrac{(X_{12} - X_{20})^2}{Q} \sim 2\, F_{1,9}$

(D) $\dfrac{P - X_{14}}{\sqrt{Q}} \sim \dfrac{\sqrt{11}}{10}\, t_9$

Answer: A; B; C

JAM 2026, Q.10 MCQ, 1 mark [8b]

Let $X_1, X_2, \ldots, X_9, Y$ be independent and identically distributed $N(\mu, \sigma^2)$ random variables. Define $\bar{X} = \frac{1}{9}\sum_{i=1}^{9} X_i$, $S^2 = \frac{1}{8}\sum_{i=1}^{9}(X_i - \bar{X})^2$. The distribution of $\frac{3}{\sqrt{10}}\left(\frac{\bar{X}-Y}{S}\right)$ is

(A) χ^2_8

(B) χ^2_9

(C) t_8

(D) t_9

Answer: C

JAM 2026, Q.30 MCQ, 2 marks [8b]

Let X_1, X_2, X_3 be three independent random variables such that $X_1 \sim N\left(0, \frac{1}{2}\right)$, $X_2 \sim N(0, 2)$, and $X_3 \sim N(0, 4)$. Consider the following statements.

P: $\frac{16X_1^2}{2X_2^2 + X_3^2}$ has F distribution with 1 and 2 degrees of freedom.

Q: $\frac{2X_1 - X_2}{2X_1 + X_2}$ has standard Cauchy distribution.

Then

(A) ***P*** is correct and ***Q*** is NOT correct

(B) ***P*** is NOT correct and ***Q*** is correct

(C) Both ***P*** and ***Q*** are correct

(D) Neither ***P*** nor ***Q*** is correct

Answer: C

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9 Estimation

9.1 9a. Unbiasedness, consistency, relative efficiency

JAM 2023, Q.19 MCQ, 2 marks [9a]

Let $X_1, X_2, \ldots, X_n$ be a random sample from a population having the probability density function

$$f(x;\mu) = \begin{cases} \dfrac{1}{2} e^{-\left(\frac{x-2\mu}{2}\right)}, & \text{if } x > 2\mu, \\ 0, & \text{otherwise,} \end{cases}$$

where $-\infty < \mu < \infty$. For estimating μ, consider estimators

$$T_1 = \frac{\bar{X} - 2}{2} \quad \text{and} \quad T_2 = \frac{nX_{(1)} - 2}{2n},$$

where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$. Then, which one of the following statements is TRUE?

(A) T_1 is consistent but T_2 is NOT consistent

(B) T_2 is consistent but T_1 is NOT consistent

(C) Both T_1 and T_2 are consistent

(D) Neither T_1 nor T_2 is consistent

Answer: C

JAM 2023, Q.44 NAT, 1 mark [9a]

Let $X_1, X_2, \ldots, X_{10}$ be a random sample from an $N(0, \sigma^2)$ distribution, where $\sigma > 0$ is an unknown parameter. For some real constant c, let $Y = \frac{c}{10}\sum_{i=1}^{10}|X_i|$ be an unbiased estimator of σ. Then, the value of c equals __________ (round off to two decimal places)

Answer: 1.24 to 1.26

JAM 2024, Q.27 MCQ, 2 marks [9a]

For $n \geq 2$, let $\epsilon_1, \epsilon_2, \ldots, \epsilon_n$ be i.i.d. random variables having the $N(0,1)$ distribution. Consider n independent random variables $Y_1, Y_2, \ldots, Y_n$ defined by

$$Y_i = \beta i + \epsilon_i, \quad i = 1, 2, \ldots, n,$$

where $\beta \in \mathbb{R}$. Define $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$, $T_1 = \frac{2\bar{Y}}{n+1}$, and $T_2 = \frac{1}{n}\sum_{i=1}^{n}\frac{Y_i}{i}$. Then which of the following statements is NOT correct?

(A) T_1 is an unbiased estimator of β

(B) T_2 is an unbiased estimator of β

(C) $\mathrm{Var}(T_1) < \mathrm{Var}(T_2)$

(D) $\mathrm{Var}(T_1) = \mathrm{Var}(T_2)$

Answer: D

JAM 2025, Q.37 MSQ (one or more correct), 2 marks [9a]

Let $X_1, X_2, \ldots, X_n$, where $n > 1$, be a random sample from a $N(\theta, \theta)$ distribution, where $\theta > 0$ is an unknown parameter. Suppose

$$T_n = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad S_n^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - T_n)^2.$$

Then which of the following statements is/are correct?

(A) $T_n S_n^2$ is a consistent estimator for θ^2

(B) $T_n^4 S_n^{-4}$ is a consistent estimator for θ^2

(C) $\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2\right)$ is a complete statistic

(D) $\sum_{i=1}^{n} X_i^2$ is a complete sufficient statistic for θ

Answer: A; B; D

JAM 2025, Q.44 NAT, 1 mark [9a]

Let $\{X_k\}_{k\geq 1}$ be a sequence of i.i.d. random variables with $\mathbb{E}(X_1) = \mu$ and $\mathrm{Var}(X_1) = \sigma^2 < \infty$. Suppose c is a constant that does not depend on n such that

$$\frac{c}{n}\sum_{i=1}^{n}(X_{2i} - X_{2i-1})^2$$

is a consistent estimator of σ^2. Then c is equal to ________

(round off to 2 decimal places)

Answer: 0.50 to 0.50

9.2 9b. Sufficiency, factorization theorem, completeness

JAM 2025, Q.26 MCQ, 2 marks [9b]

Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be a random sample from a $N_2(0, 0, 1, 1, \rho)$ distribution, where ρ is an unknown parameter. Which of the following statements is NOT correct?

(A) $\left(\sum_{i=1}^n X_i^2, \sum_{i=1}^n Y_i^2, \sum_{i=1}^n X_i Y_i\right)$ is a sufficient statistic for ρ

(B) $\left(\sum_{i=1}^n X_i^2, \sum_{i=1}^n Y_i^2, \sum_{i=1}^n X_i Y_i\right)$ is not a minimal sufficient statistic for ρ

(C) $\sum_{i=1}^n X_i^2$ is an ancillary statistic

(D) $\left(\sum_{i=1}^n X_i^2, \sum_{i=1}^n Y_i^2\right)$ is an ancillary statistic

Answer: D

JAM 2025, Q.38 MSQ (one or more correct), 2 marks [9b]

Let $X_1, X_2, \ldots, X_n$, where $n > 1$, be a random sample from a continuous distribution with probability density function

$$f(x; \theta) = \begin{cases} \theta x^{\theta-1} & \text{if } 0 < x < 1, \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is an unknown parameter. Then which of the following statistics is/are sufficient for θ?

(A) $(X_1, X_2, \ldots, X_n)$

(B) $\left(X_{(1)}, X_{(2)}, \ldots, X_{(n)}\right)$, where $X_{(r)}$ is the r^{th} order statistic, $r = 1, \ldots, n$

(C) $\sum_{i=1}^n X_i$

(D) $\prod_{i=1}^n X_i$

Answer: A; B; D

JAM 2026, Q.7 MCQ, 1 mark [9b]

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n $(n \geq 2)$ from a $N(0, \sigma^2)$ distribution, where $\sigma > 0$ is an unknown parameter. Which one of the following statements is true?

(A) $\sum_{i=1}^n X_i$ is a sufficient statistic for σ^2

(B) $\sum_{i=1}^n X_i^2$ is a sufficient statistic for σ^2

(C) $\sum_{i=1}^n X_i$ is a complete statistic

(D) $\frac{1}{n}\sum_{i=1}^n X_i^2$ is a biased estimator of σ^2

Answer: B

9.3 9c. UMVUE, Rao–Blackwell, Lehmann–Scheffé, Cramér–Rao

JAM 2022, Q.9 MCQ, 1 mark [9c]

Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from $Exp\left(\frac{1}{\theta}\right)$ distribution, where $\theta > 0$ is unknown. If $\overline{X} = \frac{1}{n}\sum_{i=1}^n X_i$, then which one of the following statements is NOT true?

(A) $\overline{X}$ is the uniformly minimum variance unbiased estimator of θ

(B) $\overline{X}^2$ is the uniformly minimum variance unbiased estimator of θ^2

(C) $\frac{n}{n+1}\overline{X}^2$ is the uniformly minimum variance unbiased estimator of θ^2

(D) $\text{Var}\left(E\left(X_n \mid \overline{X}\right)\right) \leq \text{Var}(X_n)$

Answer: B

JAM 2023, Q.18 MCQ, 2 marks [9c]

Let X_1, X_2, X_3, X_4 be a random sample of size 4 from an $N(\theta, 1)$ distribution, where $\theta \in \mathbb{R}$ is an unknown parameter. Let $\bar{X} = \frac{1}{4}\sum_{i=1}^{4} X_i$, $g(\theta) = \theta^2 + 2\theta$ and $L(\theta)$ be the Cramer-Rao lower bound on variance of unbiased estimators of $g(\theta)$. Then, which one of the following statements is FALSE?

(A) $L(\theta) = (1+\theta)^2$

(B) $\bar{X} + e^{\bar{X}}$ is a sufficient statistic for θ

(C) $(1+\bar{X})^2$ is the uniformly minimum variance unbiased estimator of $g(\theta)$

(D) $\text{Var}\left((1+\bar{X})^2\right) \geq \frac{(1+\theta)^2}{2}$

Answer: C

JAM 2023, Q.55 NAT, 2 marks [9c]

Let $x_1 = 2$, $x_2 = 5$ and $x_3 = 4$ be the observed values of a random sample from a population having a probability mass function

$$f(x;\theta) = \begin{cases} \theta(1-\theta)^x, & \text{if } x = 0, 1, 2, \dots, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in (0, 1)$ is an unknown parameter. If $\hat{\tau}$ is the uniformly minimum variance unbiased estimate of θ^2, then $156\,\hat{\tau}$ equals __________

Answer: 2 to 2

JAM 2024, Q.50 NAT, 1 mark [9c]

Suppose that the lifetimes (in months) of bulbs manufactured by a company have an $\text{Exp}(\lambda)$ distribution, where $\lambda > 0$. A random sample of size 10 taken from the bulbs manufactured by the company yields the sample mean lifetime $\bar{x} = 3.52$ months. Then the uniformly minimum variance unbiased estimate of $\frac{1}{\lambda}$ based on this sample is equal to __________ months (round off to 2 decimal places)

Answer: 3.50 to 3.55

JAM 2025, Q.27 MCQ, 2 marks [9c]

Let $(Y_1, Y_2, Y_3) \in \{0, 1, \dots, n\}^3$ be a discrete random vector having joint probability mass function

$$\mathbb{P}(Y_1 = y_1, Y_2 = y_2, Y_3 = y_3) = \begin{cases} \dfrac{n!}{y_1!\,y_2!\,y_3!}\, p^{y_1}(2p)^{y_2}(1-3p)^{y_3} & \text{if } y_1 + y_2 + y_3 = n, \\ 0 & \text{otherwise,} \end{cases}$$

where $0 \leq p \leq \frac{1}{3}$ is an unknown parameter. Assume the convention $0^0 = 1$. The maximum likelihood estimator of p is denoted by $\hat{p}$. Which of the following statements is correct?

(A) $\mathbb{E}(\hat{p}) > p$

(B) $\hat{p}$ is an unbiased estimator of p, but not the uniformly minimum variance unbiased estimator of p

(C) $\hat{p}$ is the uniformly minimum variance unbiased estimator of p

(D) $\mathbb{E}(\hat{p}) < p$

Answer: C

JAM 2025, Q.57 NAT, 2 marks [9c]

Let X_1, X_2, X_3, X_4 be a random sample from a continuous distribution with probability density function

$$f(x;\theta) = \begin{cases} 2\theta^2 x^{-3} & \text{if } \theta < x < \infty, \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is an unknown parameter. It is known that $X_{(1)} = \min\{X_1, X_2, X_3, X_4\}$ is a complete sufficient statistic for θ. If the observed values are $x_1 = 15$, $x_2 = 11$, $x_3 = 10$, $x_4 = 17$, the uniformly minimum variance unbiased estimate of θ^2 is equal to ________
(answer in integer)

Answer: 75 to 75

JAM 2026, Q.37 MSQ (one or more correct), 2 marks [9c]

Let X_1, X_2 be a random sample of size 2 from an $\text{Exp}(\theta)$ distribution, where $\theta > 0$ is an unknown parameter. Let

$$T_1 = \begin{cases} 1, & X_1 > 2, \\ 0, & \text{otherwise,} \end{cases} \qquad \text{and} \qquad T_2 = \max\left\{0, \ \frac{X_1 + X_2 - 2}{X_1 + X_2}\right\}.$$

Which of the following statements is/are true?

(A) T_1 is an unbiased estimator of $e^{-2/\theta}$

(B) T_2 is the uniformly minimum variance unbiased estimator of $e^{-2/\theta}$

(C) $\text{Var}(T_1) \leq \frac{2}{\theta} e^{-4/\theta}$ for all $\theta > 0$

(D) (X_1, X_2) is a complete sufficient statistic

Answer: A; B

9.4 9d. Method of moments, maximum likelihood, invariance

JAM 2022, Q.25 MCQ, 2 marks [9d]

Let $0.2, 1.2, 1.4, 0.3, 0.9, 0.7$ be the observed values of a random sample of size 6 from a continuous distribution with the probability density function

$$f(x) = \begin{cases} 1, & 0 < x \leq \frac{1}{2} \\ \dfrac{1}{2\theta - 1}, & \frac{1}{2} < x \leq \theta \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > \frac{1}{2}$ is unknown. Then the maximum likelihood estimate and the method of moments estimate of θ, respectively, are

(A) $\frac{7}{5}$ and 2

(B) $\frac{47}{60}$ and $\frac{32}{15}$

(C) $\frac{7}{5}$ and $\frac{32}{15}$

(D) $\frac{7}{5}$ and $\frac{47}{60}$

Answer: C

JAM 2022, Q.39 MSQ (one or more correct), 2 marks [9d]

Let X_1, X_2, X_3, X_4 be a random sample from a continuous distribution with the probability density function $f(x) = \frac{1}{2}e^{-|x-\theta|}$, $x \in \mathbb{R}$, where $\theta \in \mathbb{R}$ is unknown. Let the corresponding order statistics be denoted by $X_{(1)} < X_{(2)} < X_{(3)} < X_{(4)}$. Then which of the following statements is/are true?

(A) $\frac{1}{2}\left(X_{(2)} + X_{(3)}\right)$ is the unique maximum likelihood estimator of θ

(B) $\left(X_{(1)}, X_{(2)}, X_{(3)}, X_{(4)}\right)$ is a sufficient statistic for θ

(C) $\frac{1}{4}\left(X_{(2)} + X_{(3)}\right)\left(X_{(2)} + X_{(3)} + 2\right)$ is a maximum likelihood estimator of $\theta(\theta + 1)$

(D) $(X_1X_2X_3,\ X_1X_2X_4)$ is a complete statistic

Answer: B; C

JAM 2022, Q.49 NAT, 1 mark [9d]

Let $2.5, -1.0, 0.5, 1.5$ be the observed values of a random sample of size 4 from a continuous distribution with the probability density function

$$f(x) = \frac{1}{8}e^{-|x-2|} + \frac{3}{4\sqrt{2\pi}}e^{-\frac{1}{2}(x-\theta)^2}, \quad x \in \mathbb{R},$$

where $\theta \in \mathbb{R}$ is unknown. Then the method of moments estimate of θ equals __________ (round off to 2 decimal places)

Answer: 0.48 to 0.52

JAM 2023, Q.20 MCQ, 2 marks [9d]

Let $X_1, X_2, \ldots, X_n$ be a random sample from a $U\left(\theta + \frac{\sigma}{\sqrt{3}}, \theta + \sqrt{3}\,\sigma\right)$ distribution, where $\theta \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $S = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$. Let $\hat{\theta}$ and $\hat{\sigma}$ be the method of moment estimators of θ and σ, respectively. Then, which one of the following statements is FALSE?

(A) $\hat{\sigma} + \sqrt{3}\,\hat{\theta} = \sqrt{3}\,\bar{X} - 3S$

(B) $2\sqrt{3}\,\hat{\sigma} + \hat{\theta} = \bar{X} - 4\sqrt{3}\,S$

(C) $\sqrt{3}\,\hat{\sigma} + \hat{\theta} = \bar{X} + \sqrt{3}\,S$

(D) $\hat{\sigma} - \sqrt{3}\,\hat{\theta} = 9S - \sqrt{3}\,\bar{X}$

Answer: B

JAM 2023, Q.54 NAT, 2 marks [9d]

Let $x_1 = 2.1$, $x_2 = 4.2$, $x_3 = 5.8$ and $x_4 = 3.9$ be the observed values of a random sample X_1, X_2, X_3 and X_4 from a population having a probability density function

$$f(x;\theta) = \begin{cases} \dfrac{x}{\theta^2}\, e^{-\frac{x}{\theta}}, & \text{if } x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in (0, \infty)$ is an unknown parameter. Then, the maximum likelihood estimate of $\mathrm{Var}(X_1)$ equals __________

Answer: 8 to 8

JAM 2024, Q.28 MCQ, 2 marks [9d]

A biased coin, with probability of head as p, is tossed m times independently. It is known that $p \in \left\{\frac{1}{4}, \frac{3}{4}\right\}$ and $m \in \{3, 5\}$. If 3 heads are observed in these m tosses, then which of the following statements is correct?

(A) $\left(3, \frac{3}{4}\right)$ is a maximum likelihood estimator of (m, p)

(B) $\left(5, \frac{1}{4}\right)$ is a maximum likelihood estimator of (m, p)

(C) $\left(5, \frac{3}{4}\right)$ is a maximum likelihood estimator of (m, p)

(D) Maximum likelihood estimator of (m, p) is NOT unique

Answer: A

JAM 2024, Q.58 NAT, 2 marks [9d]

Let $X_1, X_2, \dots, X_{10}$ be a random sample from a $U(-\theta, \theta)$ distribution, where $\theta \in (0, \infty)$. Let $X_{(10)} = \max\{X_1, X_2, \dots, X_{10}\}$ and $X_{(1)} = \min\{X_1, X_2, \dots, X_{10}\}$. If the observed values of $X_{(10)}$ and $X_{(1)}$ are 8 and -10, respectively, then the maximum likelihood estimate of θ is equal to __________ (answer in integer)

Answer: 10 to 10

JAM 2025, Q.40 MSQ (one or more correct), 2 marks [9d]

Let $Y_1, Y_2, \dots, Y_n$ be i.i.d. discrete random variables from a population with probability mass function

$$\mathbb{P}(Y = y; \theta) = \begin{cases} \theta(1-\theta)^y & \text{if } y \in \mathbb{N} \cup \{0\}, \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < \theta \le 1$ is an unknown parameter. Assume the convention $0^0 = 1$. If $\hat{\theta}$ is the method of moments estimator of θ, then which of the following statements is/are correct?

(A) $\hat{\theta}$ is also the maximum likelihood estimator of θ

(B) $\hat{\theta}$ is an unbiased estimator of θ

(C) $\hat{\theta}$ is a consistent estimator of θ

(D) $\dfrac{1}{\hat{\theta}}$ is an unbiased estimator of $\dfrac{1}{\theta}$

Answer: A; C; D

JAM 2025, Q.46 NAT, 1 mark [9d]

Let $(X_1, Y_1), (X_2, Y_2), \dots, (X_{100}, Y_{100})$ be i.i.d. discrete random vectors each having joint probability mass function

$$\mathbb{P}(X = x, Y = y) = \frac{e^{-(1+x)\lambda}\left((1+x)\lambda\right)^y}{y!} p^x (1-p)^{1-x}, \quad x \in \{0, 1\}, \quad y \in \mathbb{N} \cup \{0\},$$

where $\lambda > 0$ and $0 < p < 1$ are unknown parameters. If the observed values of $\sum_{i=1}^{100} X_i$ and $\sum_{i=1}^{100} Y_i$ are 54 and 521 respectively, the maximum likelihood estimate of λ is equal to ________
(round off to 2 decimal places)

Answer: 3.38 to 3.39

JAM 2025, Q.54 NAT, 2 marks [9d]

Suppose $X_1, X_2, \dots, X_{10}, Y_1, Y_2, \dots, Y_{10}$ are independent random variables, where $X_i \sim N(0, \sigma^2)$ and $Y_i \sim N(0, 3\sigma^2)$ for $i = 1, 2, \dots, 10$. The observables are $D_1, \dots, D_{10}$, where D_i denotes the Euclidean distance between the points $(X_i, Y_i, 0)$ and $(0, 0, 5)$ for $i = 1, 2, \dots, 10$. If the observed value of $\sum_{i=1}^{10} D_i^2$ is equal to 1050, then the method of moments estimate of σ^2 is equal to ________
(answer in integer)

Answer: 20 to 20

JAM 2026, Q.24 MCQ, 2 marks [9d]

Let $X_1, X_2, \dots, X_n$ be a random sample of size n $(n \ge 1)$ from a distribution with the probability density function

$$f(x) = \frac{1}{2\beta^3} e^{3x - \frac{e^x}{\beta}}, \qquad x \in \mathbb{R},$$

where $\beta > 0$ is an unknown parameter. Which one of the following is a maximum likelihood estimator of β?

(A) $\frac{3}{n}\sum_{i=1}^{n} e^{X_i}$

(B) $\frac{1}{n}\sum_{i=1}^{n} X_i$

(C) $\frac{1}{3n}\sum_{i=1}^{n} e^{X_i}$

(D) $\frac{1}{n}\sum_{i=1}^{n} X_i^3$

Answer: C

JAM 2026, Q.25 MCQ, 2 marks [9d]

Let $X_1, X_2, \ldots, X_{10}$ be a random sample of size 10 from an $\text{Exp}(\beta)$ distribution, where $\beta > 0$ is an unknown parameter. Let $T = \frac{1}{10}\sum_{i=1}^{10} X_i$. Consider the following statements.

$\boldsymbol{P}$: $(\log_e 2)T$ is a maximum likelihood estimator of median of $X_{(1)}$.

$\boldsymbol{Q}$: $e^{-\frac{2}{T}}$ is a maximum likelihood estimator of $P(X_1 < 2)$.

Then

(A) $\boldsymbol{P}$ is correct and $\boldsymbol{Q}$ is NOT correct

(B) $\boldsymbol{P}$ is NOT correct and $\boldsymbol{Q}$ is correct

(C) Both $\boldsymbol{P}$ and $\boldsymbol{Q}$ are correct

(D) Neither $\boldsymbol{P}$ nor $\boldsymbol{Q}$ is correct

Answer: D

JAM 2026, Q.48 NAT, 1 mark [9d]

Let X_1, X_2, X_3, X_4 be a random sample of size 4 from a distribution with the probability density function

$$f(x) = \begin{cases} \alpha(\alpha+1)x^{\alpha-1}(1-x), & 0 < x < 1, \\ 0, & \text{otherwise,} \end{cases}$$

where $\alpha > 0$ is an unknown parameter. Based on the observed data 0.2, 0.1, 0.3, 0.4, the method of moments estimate of α equals __________ (rounded off to two decimal places)

Answer: 0.66 to 0.68

9.5 9e. Least squares and simple linear regression

JAM 2022, Q.59 NAT, 2 marks [9e]

Consider the linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, $i = 1, 2, \ldots, 6$, where β_0 and β_1 are unknown parameters and ϵ_i's are independent and identically distributed random variables having $N(0,1)$ distribution. The data on (x_i, y_i) are given in the following table.

x_i	1.0	2.0	2.5	3.0	3.5	4.5
y_i	2.0	3.0	3.5	4.2	5.0	5.4

If $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are the least squares estimates of β_0 and β_1, respectively, based on the above data, then $\widehat{\beta_0} + \widehat{\beta_1}$ equals __________ (round off to 2 decimal places)

Answer: 2.00 to 2.10

JAM 2023, Q.40 MSQ (one or more correct), 2 marks [9e]

Based on 10 data points $(x_1, y_1), (x_2, y_2), \ldots, (x_{10}, y_{10})$ on a variable (X, Y), the simple regression lines of Y on X and X on Y are obtained as $2y - x = 8$ and $y - x = -3$, respectively. Let $\bar{x} = \frac{1}{10}\sum_{i=1}^{10} x_i$ and $\bar{y} = \frac{1}{10}\sum_{i=1}^{10} y_i$. Then, which of the following statements is/are TRUE?

(A) $\sum_{i=1}^{10} x_i = 140$

(B) $\sum_{i=1}^{10} y_i = 110$

(C) $\dfrac{\sum_{i=1}^{10}(x_i - \bar{x})\, y_i}{\sqrt{\left(\sum_{i=1}^{10}(x_i - \bar{x})^2\right)\left(\sum_{i=1}^{10}(y_i - \bar{y})^2\right)}} = -\dfrac{1}{\sqrt{2}}$

(D) $\dfrac{\sum_{i=1}^{10}(x_i - \bar{x})^2}{\sum_{i=1}^{10}(y_i - \bar{y})^2} = 2$

Answer: A; B; D

JAM 2025, Q.39 MSQ (one or more correct), 2 marks [9e]

A simple linear regression model $Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, with $x_i = (-1)^i$ for $i = 1, 2, \ldots, 20$, is fitted. The random error variables ϵ_i are uncorrelated with mean 0 and finite variance $\sigma^2 > 0$. Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be the least squares estimators of β_0 and β_1 respectively. Let $\hat{Y}_i$ be the fitted value of the i^{th} response variable Y_i, for $i = 1, \ldots, 20$. Which of the following statements is/are correct?

(A) $\operatorname{Cov}\left(\hat{\beta}_0, \hat{\beta}_1\right) = 0$

(B) $\operatorname{Var}\left(\hat{\beta}_0\right) = \operatorname{Var}\left(\hat{\beta}_1\right)$

(C) $\operatorname{Var}\left(\hat{\beta}_0\right) = \operatorname{Cov}\left(\hat{Y}_i, \hat{\beta}_0\right)$ for all $i = 1, \ldots, 20$

(D) $\operatorname{Var}\left(\hat{\beta}_1\right) = \operatorname{Cov}\left(\hat{Y}_i, \hat{\beta}_1\right)$ for all $i = 1, \ldots, 20$

Answer: A; B; C

JAM 2026, Q.38 MSQ (one or more correct), 2 marks [9e]

Consider a simple linear regression model

$$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \qquad i = 1, 2, 3, 4, 5,$$

where the random errors ϵ_i have mean 0 and variance 36, and are uncorrelated. Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be the least squares estimators of β_0 and β_1, respectively. The above model is fitted to the following dataset

x	-2	-1	0	1	2
y	2.7	a	-0.1	b	-2.3

where $(a, b) \in \mathbb{R}^2$. Which of the following statements is/are true?

(A) If the observed value of $(\hat{\beta}_0, \hat{\beta}_1)$ is $(0, -0.11)$, then $(a, b) = (-4.6, 4.3)$

(B) If $(a, b) = (0, 1)$, then the observed value of $(\hat{\beta}_0, \hat{\beta}_1)$ is $(0.26, 0.20)$

(C) $\operatorname{Cov}(\bar{Y}, \hat{\beta}_0) = 7.2$, where $\bar{Y} = \frac{1}{5}\sum_{i=1}^{5} Y_i$

(D) $\operatorname{Var}(\hat{\beta}_0) = 2\operatorname{Var}(\hat{\beta}_1)$

Answer: A; C; D

9.6 9f. Confidence intervals

JAM 2022, Q.28 MCQ, 2 marks [9f]

Let $X_1, X_2, \ldots, X_{16}$ be a random sample from a $N(4\mu, 1)$ distribution and $Y_1, Y_2, \ldots, Y_8$ be a random sample from a $N(\mu, 1)$ distribution, where $\mu \in \mathbb{R}$ is unknown. Assume that the two random samples are independent. If you are looking for a confidence interval for μ based on the statistic $8\overline{X} + \overline{Y}$, where $\overline{X} = \frac{1}{16}\sum_{i=1}^{16} X_i$ and $\overline{Y} = \frac{1}{8}\sum_{i=1}^{8} Y_i$, then which one of the following statements is true?

(A) There exists a 90% confidence interval for μ of length less than 0.1

(B) There exists a 90% confidence interval for μ of length greater than 0.3

(C) $\left[\frac{8\overline{X}+\overline{Y}}{33}-\frac{1.645}{2\sqrt{66}}, \frac{8\overline{X}+\overline{Y}}{33}+\frac{1.645}{2\sqrt{66}}\right]$ is the unique 90% confidence interval for μ

(D) μ always belongs to its 90% confidence interval

Answer: B

JAM 2023, Q.33 MSQ (one or more correct), 2 marks [9f]

Let $x_1, x_2, \ldots, x_{10}$ be the observed values of a random sample of size 10 from an $N(\theta, \sigma^2)$ distribution, where $\theta \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. If

$$\bar{x} = \frac{1}{10}\sum_{i=1}^{10} x_i = 0 \quad \text{and} \quad s = \sqrt{\frac{1}{9}\sum_{i=1}^{10}(x_i-\bar{x})^2} = 2,$$

then based on the values of $\bar{x}$ and s and using Student's t-distribution with 9 degrees of freedom, 90% confidence interval(s) for θ is/are

(A) $(-0.8746,\ \infty)$

(B) $(-0.8746,\ 0.8746)$

(C) $(-1.1587,\ 1.1587)$

(D) $(-\infty,\ 0.8746)$

Answer: A; C; D

JAM 2024, Q.59 NAT, 2 marks [9f]

Suppose that the weights (in kgs) of six months old babies, monitored at a healthcare facility, have $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. Let $X_1, X_2, \ldots, X_9$ be a random sample of the weights of such babies. Let $\bar{X} = \frac{1}{9}\sum_{i=1}^{9} X_i$, $S = \sqrt{\frac{1}{8}\sum_{i=1}^{9}(X_i-\bar{X})^2}$ and let a 95% confidence interval for μ based on t-distribution be of the form

$$\left(\bar{X}-h(S),\ \bar{X}+h(S)\right),$$

for an appropriate function h of random variable S. If the observed values of $\bar{X}$ and S^2 are 9 and 9.5, respectively, then the width of the confidence interval is equal to __________ (round off to 2 decimal places)

(You may use $t_{9,0.025} = 2.262$, $t_{8,0.025} = 2.306$, $t_{9,0.05} = 1.833$, $t_{8,0.05} = 1.86$)

Answer: 4.70 to 4.80

JAM 2025, Q.10 MCQ, 1 mark [9f]

Let X be a single sample from a continuous distribution with probability density function

$$f(x;\theta) = \begin{cases} \dfrac{2(\theta-x)}{\theta^2} & \text{if } 0 < x < \theta, \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is an unknown parameter. For $0 < \alpha < 0.05$, a $100(1-\alpha)\%$ confidence interval for θ based on X is

(A) $\left(\dfrac{X}{1-\sqrt{\dfrac{\alpha}{2}}}, \dfrac{X}{1-\sqrt{1-\dfrac{\alpha}{2}}}\right)$

(B) $\left(\dfrac{X}{1-\sqrt{\alpha}},\ \dfrac{X}{1-\sqrt{1-\alpha}}\right)$

(C) $\left(\left(1-\sqrt{1-\dfrac{\alpha}{2}}\right)X,\ \left(1-\sqrt{\dfrac{\alpha}{2}}\right)X\right)$

(D) $\left(\dfrac{\alpha}{2}X,\ \left(1-\dfrac{\alpha}{2}\right)X\right)$

Answer: A

JAM 2026, Q.57 NAT, 2 marks [9f]

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n $(n > 1)$ from a $N(\mu, 1)$ distribution, where $\mu \in \mathbb{R}$ is an unknown parameter. If $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, then the minimum value of n such that $P(2\bar{X} - 1 < 2\mu < 2\bar{X} + 1) \geq 0.99$ equals __________ (in integer)

[You may use $\Phi(2.58) = 0.9951$, $\Phi(2.48) = 0.9934$]

Answer: 27 to 27

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10 Testing of Hypotheses

10.1 10a. Type I/II errors, size, power, p-value

JAM 2022, Q.10 MCQ, 1 mark [10a]

Let $X_1, X_2, \ldots, X_n$ $(n \geq 3)$ be a random sample from a $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$ are both unknown. Then which one of the following is a simple null hypothesis?

(A) $H_0 : \mu < 5,\ \sigma^2 = 3$

(B) $H_0 : \mu = 5,\ \sigma^2 > 3$

(C) $H_0 : \mu = 5,\ \sigma^2 = 3$

(D) $H_0 : \mu = 5$

Answer: C

JAM 2022, Q.29 MCQ, 2 marks [10a]

Let X_1, X_2, X_3, X_4 be a random sample from a distribution with the probability mass function

$$f(x) = \begin{cases} \theta^x (1-\theta)^{1-x}, & x = 0, 1 \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta \in (0,1)$ is unknown. Let $0 < \alpha \leq 1$. To test the hypothesis $H_0 : \theta = \frac{1}{2}$ against $H_1 : \theta > \frac{1}{2}$, consider the size α test that rejects H_0 if and only if $\sum_{i=1}^{4} X_i \geq k_\alpha$, for some $k_\alpha \in \{0, 1, 2, 3, 4\}$. Then for which one of the following values of α, the size α test does NOT exist?

(A) $\frac{1}{16}$

(B) $\frac{1}{4}$

(C) $\frac{11}{16}$

(D) $\frac{5}{16}$

Answer: B

JAM 2022, Q.30 MCQ, 2 marks [10a]

Let X_1, X_2, X_3, X_4 be a random sample from a Poisson distribution with unknown mean $\lambda > 0$. For testing the hypothesis

$$H_0 : \lambda = 1 \text{ against } H_1 : \lambda = 1.5,$$

let β denote the power of the test that rejects H_0 if and only if $\sum_{i=1}^{4} X_i \geq 5$. Then which one of the following statements is true?

(A) $\beta > 0.80$

(B) $0.75 < \beta \leq 0.80$

(C) $0.70 < \beta \leq 0.75$

(D) $0.65 < \beta \leq 0.70$

Answer: C

JAM 2022, Q.40 MSQ (one or more correct), 2 marks [10a]

Let $X_1, X_2, \ldots, X_n$ $(n > 1)$ be a random sample from a $N(\mu, 1)$ distribution, where $\mu \in \mathbb{R}$ is unknown. Let $0 < \alpha < 1$. To test the hypothesis $H_0 : \mu = 0$ against $H_1 : \mu = \delta$, where $\delta > 0$ is a constant, let β denote the power of the size α test that rejects H_0 if and only if $\frac{1}{n}\sum_{i=1}^{n} X_i > c_\alpha$, for some constant c_α. Then which of the following statements is/are true?

(A) For a fixed value of δ, β increases as α increases

(B) For a fixed value of α, β increases as δ increases

(C) For a fixed value of δ, β decreases as α increases

(D) For a fixed value of α, β decreases as δ increases

Answer: A; B

JAM 2022, Q.50 NAT, 1 mark [10a]

Let $X_1, X_2, \ldots, X_{25}$ be a random sample from a $N(\mu, 1)$ distribution, where $\mu \in \mathbb{R}$ is unknown. Consider testing of the hypothesis $H_0 : \mu = 5.2$ against $H_1 : \mu = 5.6$. The null hypothesis is rejected if and only if $\frac{1}{25}\sum_{i=1}^{25} X_i > k$, for some constant k. If the size of the test is 0.05, then the probability of type-II error equals __________ (round off to 2 decimal places)

Answer: 0.34 to 0.38

JAM 2023, Q.50 NAT, 1 mark [10a]

Let X_1, X_2 be a random sample from a distribution having a probability density function

$$f(x; \theta) = \begin{cases} \dfrac{1}{\theta}\, e^{-\frac{x}{\theta}}, & \text{if } x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in (0, \infty)$ is an unknown parameter. For testing the null hypothesis $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$, consider a test that rejects H_0 for small observed values of the statistic $W = \frac{X_1 + X_2}{2}$. If the observed values of X_1 and X_2 are 0.25 and 0.75, respectively, then the p-value equals __________ (round off to two decimal places)

Answer: 0.25 to 0.27

JAM 2023, Q.58 NAT, 2 marks [10a]

Let X_1, X_2 be a random sample from a $U(0, \theta)$ distribution, where $\theta > 0$ is an unknown parameter. For testing the null hypothesis $H_0 : \theta \in (0, 1] \cup [2, \infty)$ against $H_1 : \theta \in (1, 2)$, consider the critical region

$$R = \left\{(x_1, x_2) \in \mathbb{R} \times \mathbb{R} : \frac{5}{4} < \max\{x_1, x_2\} < \frac{7}{4}\right\}.$$

Then, the size of the critical region equals __________

Answer: 0.375 to 0.375

JAM 2023, Q.59 NAT, 2 marks [10a]

Let $X_1, X_2, \ldots, X_5$ be a random sample from a $\mathrm{Bin}(1, \theta)$ distribution, where $\theta \in (0, 1)$ is an unknown parameter. For testing the null hypothesis $H_0 : \theta \leq 0.5$ against $H_1 : \theta > 0.5$, consider the two tests T_1 and T_2 defined as:
T_1: Reject H_0 if, and only if, $\sum_{i=1}^{5} X_i = 5$.
T_2: Reject H_0 if, and only if, $\sum_{i=1}^{5} X_i \geq 3$.
Let β_i be the probability of making Type-II error, at $\theta = \frac{2}{3}$, when the test T_i, $i = 1, 2$, is used. Then, the value of $\beta_1 + \beta_2$ equals __________ (round off to two decimal places)

Answer: 1.07 to 1.09

JAM 2024, Q.30 MCQ, 2 marks [10a]

Let $X_1, X_2, \ldots, X_{10}$ be a random sample from a $N(0, \sigma^2)$ distribution, where $\sigma > 0$ is unknown. For testing $H_0\colon \sigma^2 \leq 1$ against $H_1\colon \sigma^2 > 1$, a test of size $\alpha = 0.05$ rejects H_0 if and only if $\sum_{i=1}^{10} X_i^2 > 18.307$. Let β be the power of this test, at $\sigma^2 = 2$. Then β lies in the interval
(You may use $\chi^2_{10,0.05} = 18.307$, $\chi^2_{10,0.1} = 15.9872$, $\chi^2_{10,0.25} = 12.5489$, $\chi^2_{10,0.5} = 9.3418$, $\chi^2_{10,0.75} = 6.7372$, $\chi^2_{10,0.9} = 4.8652$, $\chi^2_{10,0.95} = 3.9403$, $\chi^2_{10,0.975} = 3.247$)

(A) $(0.50, 0.75)$

(B) $(0.75, 0.90)$

(C) $(0.90, 0.95)$

(D) $(0.95, 0.975)$

Answer: A

JAM 2024, Q.60 NAT, 2 marks [10a]

Let X_1, X_2, X_3 be a random sample from a Poisson distribution with mean λ, $\lambda > 0$. For testing $H_0\colon \lambda = \frac{1}{8}$ against $H_1\colon \lambda = 1$, a test rejects H_0 if and only if $X_1 + X_2 + X_3 > 1$. The power of this test is equal to

____

(round off to 2 decimal places)

Answer: 0.79 to 0.81

JAM 2025, Q.45 NAT, 1 mark [10a]

Let $X_1, X_2, \ldots, X_{10}$ be i.i.d. $\mathrm{U}(0, \theta)$ random variables, where $\theta > 0$ is unknown. For testing the null hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H_1 : \theta = 0.9$, consider a test that rejects H_0 if

$$X_{(10)} = \max\{X_1, X_2, \ldots, X_{10}\} < 0.8.$$

Then the probability of type I error of the test is equal to ________
(round off to 2 decimal places)

Answer: 0.10 to 0.11

JAM 2026, Q.26 MCQ, 2 marks [10a]

Let X_1, X_2, X_3, X_4 be a random sample of size 4 from a χ^2_m distribution, where $m \in \mathbb{N}$ is an unknown parameter. To test $H_0 : m = 1$ against $H_1 : m = 2$, the critical region $\sum_{i=1}^{4} X_i > 6$ is being used. If α and β denote the probabilities of Type-I error and Type-II error, respectively, then which one of the following statements is true?
[You may use $\chi^2_{1,0.0143} = \chi^2_{2,0.0498} = \chi^2_{4,0.1991} = \chi^2_{8,0.6472} = 6$]

(A) $0.20 < \frac{3}{4}\alpha + \frac{1}{4}\beta < 0.25$

(B) $\alpha > 0.20$

(C) $\beta < 0.30$

(D) The power of the test lies in the interval $\left(0, \frac{1}{2}\right)$

Answer: A

JAM 2026, Q.27 MCQ, 2 marks [10a]

Let X be a random sample of size 1 from a $U(0, \theta)$ distribution, where $\theta > 0$ is an unknown parameter. To test $H_0 : \theta \geq 1$ against $H_1 : \theta < 1$, the critical region $X < \frac{3}{4}$ is being used. If $\beta(\cdot)$ is the power function of the test, then which one of the following statements is NOT true?

(A) The size of the test is $\frac{3}{4}$

(B) $\inf_{\theta<1} \beta(\theta) = \frac{3}{4}$

(C) For all $\theta > 0$, $\beta(\theta) < 1$

(D) $\lim_{\theta \to 0} \beta(\theta) = 1$

Answer: C

JAM 2026, Q.58 NAT, 2 marks [10a]

Let X_1, X_2 be a random sample of size 2 from a distribution with the probability density function

$$f(x) = \begin{cases} \alpha\, x^{\alpha-1}, & 0 < x < 1, \\ 0, & \text{otherwise}, \end{cases}$$

where $\alpha > 0$ is an unknown parameter. To test $H_0 : \alpha = 1$ against $H_1 : \alpha = 2$, the most powerful test is being used. If the observed value of the random sample is $\frac{1}{4}$, $\frac{1}{2}$, then the p-value of the test equals __________ (rounded off to three decimal places)

Answer: 0.613 to 0.617

10.2 10b. Neyman–Pearson lemma, MP and UMP tests

JAM 2023, Q.10 MCQ, 1 mark [10b]

Let X be a random variable having a probability density function

$$f(x;\theta) = \begin{cases} (3-\theta)\, x^{2-\theta}, & \text{if } 0 < x < 1, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta \in \{0, 1\}$. For testing the null hypothesis $H_0 : \theta = 0$ against $H_1 : \theta = 1$, the power of the most powerful test, at the level of significance $\alpha = 0.125$, equals

(A) 0.15

(B) 0.25

(C) 0.35

(D) 0.45

Answer: B

JAM 2024, Q.29 MCQ, 2 marks [10b]

Let $X_1, X_2, \ldots, X_n$ be a random sample from an $\mathrm{Exp}(\lambda)$ distribution, where $\lambda \in \{1, 2\}$. For testing $H_0 \colon \lambda = 1$ against $H_1 \colon \lambda = 2$, the most powerful test of size α, $\alpha \in (0, 1)$, will reject H_0 if and only if

(A) $\sum_{i=1}^{n} X_i \leq \frac{1}{2}\chi^2_{2n,1-\alpha}$

(B) $\sum_{i=1}^{n} X_i \geq 2\,\chi^2_{2n,1-\alpha}$

(C) $\sum_{i=1}^{n} X_i \leq \frac{1}{2}\chi^2_{n,1-\alpha}$

(D) $\sum_{i=1}^{n} X_i \geq 2\,\chi^2_{n,1-\alpha}$

Answer: A

JAM 2024, Q.40 MSQ (one or more correct), 2 marks [10b]

Let θ_0 and θ_1 be real constants such that $\theta_1 > \theta_0$. Suppose that a random sample is taken from a $N(\theta, 1)$ distribution, $\theta \in \mathbb{R}$. For testing $H_0\colon \theta = \theta_0$ against $H_1\colon \theta = \theta_1$ at level 0.05, let α and β denote the size and the power, respectively, of the most powerful test, ψ_0. Then which of the following statements is/are correct?

(A) $\beta < \alpha$

(B) The test ψ_0 is the uniformly most powerful test of level α for testing $H_0\colon \theta = \theta_0$ against $H_1\colon \theta > \theta_0$

(C) $\alpha < \beta$

(D) The test ψ_0 is the uniformly most powerful test of level α for testing $H_0\colon \theta = \theta_0$ against $H_1\colon \theta < \theta_0$

Answer: B; C

JAM 2025, Q.5 MCQ, 1 mark [10b]

Let X_1, X_2, X_3 be i.i.d. $\text{Bin}(1, \theta)$ random variables. Consider the problem of testing the null hypothesis $H_0 : \theta = \frac{1}{2}$ against the alternative hypothesis $H_1 : \theta = \frac{1}{4}$ based on X_1, X_2, X_3. Then the power of the most powerful test of size 0.125 is

(A) 0

(B) $\dfrac{1}{64}$

(C) $\dfrac{27}{64}$

(D) $\dfrac{7}{8}$

Answer: C

JAM 2025, Q.8 MCQ, 1 mark [10b]

Let X be a continuous random variable with probability density function $f(x)$. Consider the problem of testing the null hypothesis

$$H_0 : f(x) = \begin{cases} 1 & \text{if } 0 < x < 1, \\ 0 & \text{otherwise,} \end{cases}$$

against the alternative hypothesis

$$H_1 : f(x) = \begin{cases} 2x & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Then the power of the most powerful size α test, where $0 < \alpha < 1$, based on a single sample, is

(A) $\alpha(1-\alpha)$

(B) $\alpha(2-\alpha)$

(C) $1-\alpha$

(D) α

Answer: B

JAM 2025, Q.22 MCQ, 2 marks [10b]

Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with probability density function

$$f(x;\theta) = \frac{1}{2\theta} e^{-\frac{|x|}{\theta}}, \quad x \in \mathbb{R},$$

where $\theta > 0$ is an unknown parameter. The critical region for the uniformly most powerful test for testing the null hypothesis $H_0 : \theta = 2$ against the alternative hypothesis $H_1 : \theta > 2$ at level α, where $0 < \alpha < 1$, is

(A) $\left\{(x_1, \ldots, x_n) \in \mathbb{R}^n : 2\sum_{i=1}^n |x_i| < \chi^2_{2n,1-\alpha}\right\}$

(B) $\left\{(x_1, \ldots, x_n) \in \mathbb{R}^n : 2\sum_{i=1}^n |x_i| > \chi^2_{2n,\alpha}\right\}$

(C) $\left\{(x_1, \ldots, x_n) \in \mathbb{R}^n : \sum_{i=1}^n |x_i| < \chi^2_{2n,1-\alpha}\right\}$

(D) $\left\{(x_1, \ldots, x_n) \in \mathbb{R}^n : \sum_{i=1}^n |x_i| > \chi^2_{2n,\alpha}\right\}$

Answer: D

JAM 2026, Q.39 MSQ (one or more correct), 2 marks [10b]

Let Y_1, Y_2 be a random sample of size 2 from a distribution with the probability mass function

$$f(y) = \begin{cases} \theta(1-\theta)^y, & y = 0, 1, 2, \ldots \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in (0, 1)$ is an unknown parameter. Let ψ be the most powerful randomized test for testing $H_0 : \theta = 0.20$ against $H_1 : \theta = 0.75$ at level 0.05. Which of the following statements is/are true?

(A) The test ψ rejects H_0 with probability 1 if the observed value of $Y_1 + Y_2$ is 0

(B) The test ψ rejects H_0 with probability 1 if the observed value of $Y_1 + Y_2$ is 1

(C) The test ψ rejects H_0 with probability $\frac{5}{16}$ if the observed value of $Y_1 Y_2$ is 1

(D) The power of the test ψ is $\frac{621}{1024}$

Answer: A; D

10.3 10c. Likelihood ratio tests

JAM 2022, Q.60 NAT, 2 marks [10c]

Let $X_1, X_2, \ldots, X_9$ be a random sample from a $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$ are unknown. Let the observed values of $\overline{X} = \frac{1}{9}\sum_{i=1}^9 X_i$ and $S^2 = \frac{1}{8}\sum_{i=1}^9 \left(X_i - \overline{X}\right)^2$ be 9.8 and 1.44, respectively. If the likelihood ratio test is used to test the hypothesis $H_0 : \mu = 8.8$ against $H_1 : \mu > 8.8$, then the p-value of the test equals __________ (round off to 3 decimal places)

Answer: 0.017 to 0.021

JAM 2023, Q.28 MCQ, 2 marks [10c]

Let x_1, x_2, x_3 and x_4 be observed values of a random sample from an $N(\theta, \sigma^2)$ distribution, where $\theta \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. Suppose that $\bar{x} = \frac{1}{4}\sum_{i=1}^4 x_i = 3.6$ and $\frac{1}{3}\sum_{i=1}^4 (x_i - \bar{x})^2 = 20.25$. For testing the null hypothesis $H_0 : \theta = 0$ against $H_1 : \theta \neq 0$, the p-value of the likelihood ratio test equals

(A) 0.712

(B) 0.208

(C) 0.104

(D) 0.052

Answer: B

JAM 2024, Q.10 MCQ, 1 mark [10c]

Let x_1, x_2, x_3, x_4 be the observed values of a random sample from a $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma \in (0, \infty)$ are unknown parameters. Let $\bar{x}$ and $s = \sqrt{\frac{1}{3}\sum_{i=1}^{4}(x_i - \bar{x})^2}$ be the observed sample mean and the sample standard deviation, respectively. For testing $H_0\colon \mu = 0$ against $H_1\colon \mu \neq 0$, the likelihood ratio test of size $\alpha = 0.05$ rejects H_0 if and only if $\frac{|\bar{x}|}{s} > k$. Then the value of k is

(A) $\frac{1}{2}t_{3,0.025}$

(B) $t_{3,0.025}$

(C) $2t_{3,0.05}$

(D) $\frac{1}{2}t_{3,0.05}$

Answer: A

JAM 2026, Q.8 MCQ, 1 mark [10c]

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n $(n \geq 2)$ from a $N(\mu, 1)$ distribution, where $\mu \in \mathbb{R}$ is an unknown parameter. Consider the problem of testing $H_0 : \mu = 5$ against $H_1 : \mu \neq 5$. Which one of the following statements is true?

(A) The power function of the likelihood ratio test at level 0.05 has a local maximum at $\mu = 5$

(B) The power function of the likelihood ratio test at level 0.05 is decreasing on $(5, \infty)$

(C) The power function of the likelihood ratio test at level 0.05 is decreasing on $(-\infty, 5)$

(D) The power function of the likelihood ratio test at level 0.05 is increasing on $(0, 5)$

Answer: C

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11 Nonparametric Methods

11.1 11a. Runs test, empirical d.f., Kolmogorov–Smirnov, sign tests, Mann–Whitney

JAM 2026, Q.28 MCQ, 2 marks [11a]

The recorded air quality index (AQI) at a place in a city for 16 consecutive days for the current year is compared with the historical average AQI. The direction of the recorded AQI is noted as '+' if it is above the historical average and as '−' if it is below the historical average. The dataset is as below:

$$+ \; + \; - \; + \; - \; - \; + \; + \; + \; + \; - \; - \; - \; + \; + \; -$$

The total number of runs, R, is being used to test
H_0: The direction of the recorded AQI is random
against
H_1: The direction of the recorded AQI is not random
Consider the following statements.
P: The observed value of total number of runs is 8.
Q: Based on the above dataset, H_0 is rejected in favor of H_1 at level 0.05.
Then which one of the following is true?
[You may use: Under H_0, with 7 negative signs and 9 positive signs, $P(R \leq 6) = 0.108$, $P(R \leq 7) = 0.231$, $P(R \leq 8) = 0.427$]

(A) ***P*** is correct and ***Q*** is NOT correct

(B) $\boldsymbol{P}$ is NOT correct and $\boldsymbol{Q}$ is correct

(C) Both $\boldsymbol{P}$ and $\boldsymbol{Q}$ are correct

(D) Neither $\boldsymbol{P}$ nor $\boldsymbol{Q}$ is correct

Answer: A

JAM 2026, Q.59 NAT, 2 marks [11a]

The observed value of a random sample of size 9 from a distribution having continuous and strictly increasing cumulative distribution function is as below:

$$1.00,\ -0.01,\ 0.70,\ 0.25,\ -1.00,\ 1.20,\ -0.33,\ 0.68,\ -2.00$$

Let M be the unknown median of the distribution. To test $H_0 : M = 0$ against $H_1 : M > 0$, the sign test is being used. If

$$\eta = \begin{cases} 1, & \text{if } H_0 \text{ is accepted at level } 0.05, \\ 0, & \text{if } H_0 \text{ is rejected at level } 0.05, \end{cases}$$

and p is the p-value of the test, then based on the above data $p + \eta$ equals __________ (rounded off to two decimal places)

Answer: 1.49 to 1.51

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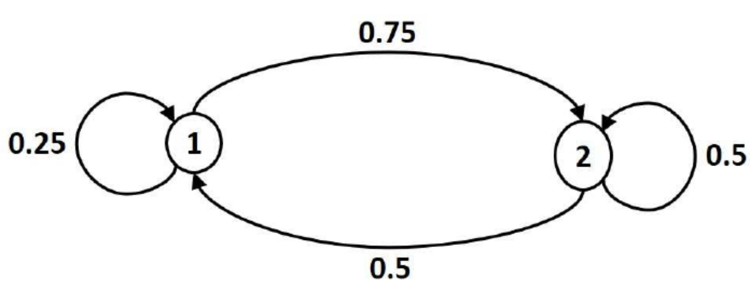
12 Stochastic Processes

12.1 12a. Discrete-time Markov chains

JAM 2026, Q.9 MCQ, 1 mark [12a]

Consider a discrete time Markov chain with state space $S = \{1, 2\}$ and the transition probability diagram shown below.
If $\pi = (\pi_1, \pi_2)$ is the stationary distribution of the Markov chain, then which one of the following statements is true?



(A) $(\pi_1, \pi_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$

(B) $(\pi_1, \pi_2) = \left(\frac{2}{5}, \frac{3}{5}\right)$

(C) $(\pi_1, \pi_2) = \left(\frac{1}{4}, \frac{3}{4}\right)$

(D) $(\pi_1, \pi_2) = \left(\frac{1}{3}, \frac{2}{3}\right)$

Answer: B

JAM 2026, Q.29 MCQ, 2 marks [12a]

Consider a discrete time Markov chain with state space $S = \{1, 2, 3\}$ and the transition probability matrix

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{5} & \frac{4}{5} \\ 0 & 1 & 0 \end{pmatrix}.$$

Which one of the following statements is true?

(A) The Markov chain is irreducible

(B) 1 is a recurrent state

(C) 2 is a transient state

(D) 3 is a recurrent state

Answer: D

12.2 12b. Poisson process

JAM 2026, Q.49 NAT, 1 mark [12b]

Let $\{N(t),\, t \geq 0\}$ be a Poisson process with rate $\lambda = 1$. Then $E[N(8)\, N(5)]$ equals __________ (in integer)

Answer: 45 to 45
